P Systems with String Objects and with Communication by Request

by

Erzsébet Csuhaj-Varjú and **György Vaszil**

Computer and Automation Research Institute, Hungarian Academy of Sciences,
H-1111 Budapest, Kende u.13-17, Hungary

E-mail:{csuhaj,vaszil}@sztaki.hu

SZTAKI - Theoretical Computer Science Research Group

Outline of the talk

- P systems with string ojects and with communication by request, definitions, example
- The two different types of communication
- The power of these constructs
- Conclusions and some further research directions

P systems in general

- A **(hierarchically embedded) structure of membranes** containig a **collection of objects** which **evolve and move** through the **regions** embraced by the membranes.

- The **evolution** of the system corresponds to a **computation**.

- There are rules associated to the regions, having two basic goals: **change** the objects (**rewriting**), **move** the objects from one membrane to some other one (**communication**).

Our model

Multisets of **strings** over sets of **terminal** and **nonterminal** symbols in each membrane, processed simultaneously by context-free **rewriting**.

Special nonterminal symbols called **query symbols** associated to each membrane. **Communication requests** are initiated by the appearance of query symbols in the processed strings.

Our model (continued)

When one or more **query symbols** are **introduced in a string**, then

- the **rewriting stops** and the
- **queries** are **satisfied** by **replacing** the query symbols with **query symbol free strings** from the **region indicated** by the query symbol, in **all** possible **combinations**.
- If **no query symbol free string** exists in the queried **region**, then the **string** containing the query **disappears**.

The result consists of all **terminal strings** which appear in an **output membrane**.

The two types of communication

Consider the **strings** as descriptions of **simple organisms** and the **query symbols** as their **"weak points"** possibly **infected** or **attacked** by other organisms. Then:

- Communication of type "i" (infection): Copies of the communicated strings are sent, one copy also **remains in the originating region**.
- Communication of type "p" (parasitism): The communicated strings **disappear from the originating region**.

An RPC system

$$\Pi = (N \cup T \cup K, \mu, (I_1, R_1), \ldots, (I_n, R_n), i_o)$$

N, T	–	finite alphabets of **nonterminal** and **terminal** symbols
$K = \{Q_1, \ldots, Q_n\}$	–	the alphabet of **query symbols**
I_i	–	finite multiset of **strings** over $N \cup T$ (the initial multiset of region i)
R_i	–	finite set of (context-free) **rules** associated to region i, of the form $A \to \alpha$, where $A \in N$, $\alpha \in (N \cup T \cup K)^*$
$i_o \in \{1, \ldots, n\}$	–	the index of the **output** membrane

Changing the objects: rewriting

$$\Pi = (N \cup T \cup K, \mu, (I_1, R_1), \ldots, (I_n, R_n), i_o)$$

$$(M_1, \ldots, M_n) \Rightarrow_{rew} (M'_1, \ldots, M'_n)$$

1. There are **no query symbols** in the strings of M_i

$M'_i = \{\{x'_1, \ldots, x'_m\}\}$ where $M_i = \{\{x_1, \ldots, x_m\}\}$ and either $x_j \Longrightarrow x'_j$ by applying a rule of R_i or $x'_j = x_j$.

Moving the objects: communication

$$(M_1, \ldots, M_n) \Rightarrow_{com} (M'_1, \ldots, M'_n)$$

2. There **are** query symbols in the strings of M_i

a) the **p-communicating** variant:

$M'_i = M_i - \boxed{M_i^{req}} - \{\{x \in M_i \mid |x|_K > 0\}\} + \bigcup_{x \in M_i, |x|_K > 0} Sat(x)$

where

$$M_i^{req} = \begin{cases} \{\{x \in M_i \mid |x|_K = 0\}\}, & \text{if there is } y \in M_j, |y|_{Q_i} > 0 \\ \emptyset, & \text{otherwise.} \end{cases}$$

$$Sat(x) = \{\{\ x_1 y_{i_1} x_2 y_{i_2} \ldots y_{i_t} x_{t+1} \mid x = x_1 Q_{i_1} x_2 Q_{i_2} \ldots Q_{i_t} x_{t+1},\ y_{i_j} \in M_{i_j}\ |y_{i_j}|_K = 0,\ 1 \le j \le t\ \}\}$$

Moving the objects: communication

$$(M_1, \ldots, M_n) \Rightarrow_{com} (M_1', \ldots, M_n')$$

2. There **are** query symbols in the strings

b) the **i-communicating** variant:

$$M_i' = M_i - \{\{x \in M_i \mid |x|_K > 0\}\} + \bigcup_{x \in M_i, |x|_K > 0} Sat(x)$$

where

$$Sat(x) = \{\{\ x_1 y_{i_1} x_2 y_{i_2} \ldots y_{i_t} x_{t+1} \mid x = x_1 Q_{i_1} x_2 Q_{i_2} \ldots Q_{i_t} x_{t+1},\ y_{i_j} \in M_{i_j} \\ |y_{i_j}|_K = 0,\ 1 \leq j \leq t\ \}\}$$

The generated language

$$\Pi = (N \cup K \cup T, \mu, (I_1, R_1), \ldots, (I_n, R_n), i_o)$$

$$L_X(\Pi) = \{x \in T^* \mid (I_1, \ldots, I_n) \Rightarrow^* (M'_1, \ldots, M'_n) \text{ and } x \in M'_{i_o}, \}$$

for $X \in \{p, i\}$.

The language classes: $pRPC_nCF$, $iRPC_nCF$

Example: A system with one membrane

$$\Pi = (\{S, A\} \cup \{Q_1\} \cup \{a, b\}, [\ ], (\{\{S, A, b\}\}, R_1), 1), \text{ with}$$

$$R_1 = \{S \to aSa, S \to Q_1, A \to A, A \to Q_1Q_1\}$$

$[S, A, b] \Rightarrow$

a) $\ldots \Rightarrow [a^nQ_1a^n, Q_1Q_1, b] \Rightarrow [a^nba^n, bb]$

b) $\ldots \Rightarrow [a^nQ_1a^n, A, b] \Rightarrow [a^nAa^n, a^nba^n] \Rightarrow \ldots \Rightarrow [a^nQ_1Q_1a^n, a^nba^n] \Rightarrow$
$[a^na^nba^na^nba^na^n]$

c) $\ldots \Rightarrow [a^nSa^n, Q_1Q_1, b] \Rightarrow [a^nSa^na^nSa^n, a^nSa^nb, ba^nSa^n, bb] \Rightarrow \ldots \Rightarrow$
$[\ldots \alpha bb\beta \ldots]$

Example: A system with one membrane

$$L(\Pi) \cap a^+ba^+ba^+ = \{a^{2n}ba^{2n}ba^{2n} \mid n \geq 1\}$$

thus, $L(\Pi) \notin CF$

$$\boxed{pRPC_1CF - CF \neq \emptyset}$$

[Csuhaj-Varjú, Pāun, Vaszil, 2006]

Tissue-like P systems and “standard” P systems

- Tissue-like P systems: **Direct communication** is possible between **any two regions** of the system.

  [Csuhaj-Varjú, Păun, Vaszil, 2006]

- Standard P systems: Direct **communication** is possible between **neighboring regions** only.

Tissue-like P systems with string objects and communication by request

$$CF \subset (tissue)XRPC_1CF \subseteq (tissue)iRPC_6CF = (tissue)pRPC_8CF = (tissue)XRPC_*CF = RE,$$

for $X \in \{i, p\}$.

[Csuhaj-Varjú, Păun, Vaszil, 2006]

In the following we will study the **standard variant**, when communication is only possible between **neighboring regions**.

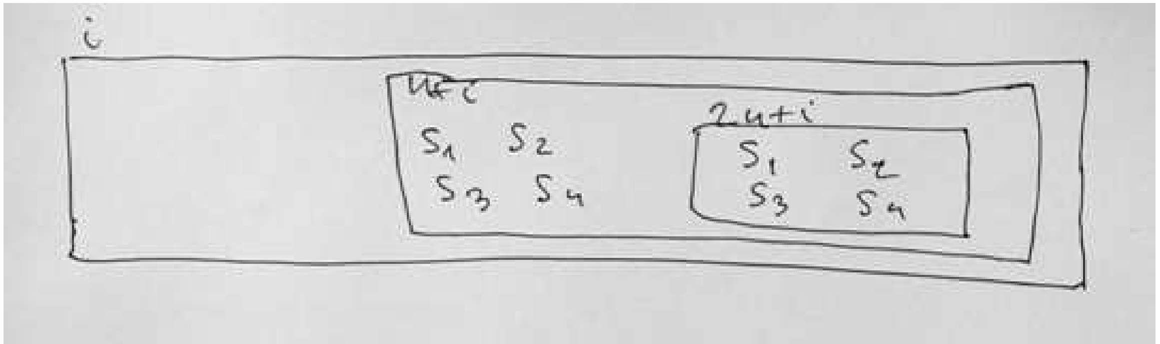
The notation $pRPC_nCF$, $iRPC_nCF$ will be used to denote the corresponding language classes.

The relationship of systems with the different types of communication

$$iRPC_nCF \subseteq pRPC_{3n}CF$$

The proof of $iRPC_nCF \subseteq pRPC_{3n}CF$

Two new regions $n+i$ and $2n+i$ are added to all regions i, with



R_{n+i}: $S_3 \to Q_{n+i},\ S_4 \to Q_{2n+i}$
R_{2n+i}: $S_1 \to Q_{n+i},\ S_2 \to Q_{2n+i}$

$$\left[\begin{array}{ll} S_1, & S_3 \\ S_2, & S_4 \end{array} \left[\begin{array}{ll} S_1, & S_3 \\ S_2, & S_4 \end{array}\right]_{2n+i}\right]_{n+i} \Rightarrow \left[\begin{array}{ll} S_1, & Q_{n+i} \\ S_2, & Q_{2n+i} \end{array} \left[\begin{array}{ll} Q_{n+i}, & S_3 \\ Q_{2n+i}, & S_4 \end{array}\right]_{2n+i}\right]_{n+i}$$

$$\Rightarrow \left[\begin{array}{ll} S_1, & S_3 \\ S_2, & S_4 \end{array} \left[\begin{array}{ll} S_1, & S_3 \\ S_2, & S_4 \end{array}\right]_{2n+i}\right]_{n+i}$$

...and so on. Region $n+i$ can always send S_1, S_2.

The proof of $iRPC_nCF \subseteq pRPC_{3n}CF$ continued

We also add

the strings: S_1, S_2,
and the rules: $S_1 \to Q_i,\ S_2 \to Q_{n+i}$

to each “original” region i.

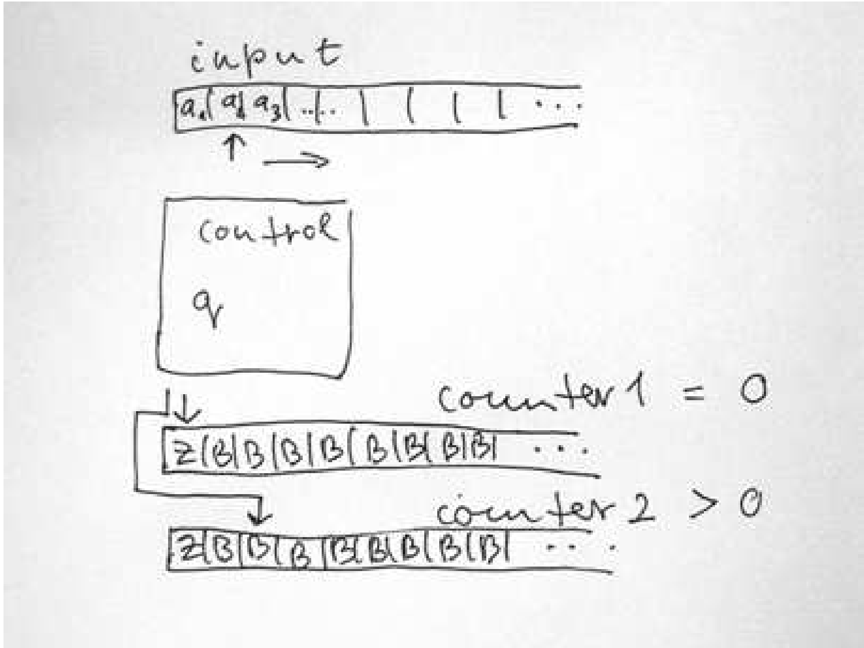
The first rule **“saves a copy”** of all strings, the second one **adds** S_1, S_2 again.

The power of RPC systems

$$\boxed{iRPC_{10}CF = RE}$$

The proof of $iRPC_{10}CF = RE$

RPC systems with i-type of communication can **simulate two-counter machines**.



Instantenous description:

$$(q, a, c_1, c_2)$$

Transition:

$$(q, a, c_1, c_2; \; q', e_1, e_2)$$

The proof of $iRPC_{10}CF = RE$ continued

Explanations how the membrane system corresponds to the two-counter machine:

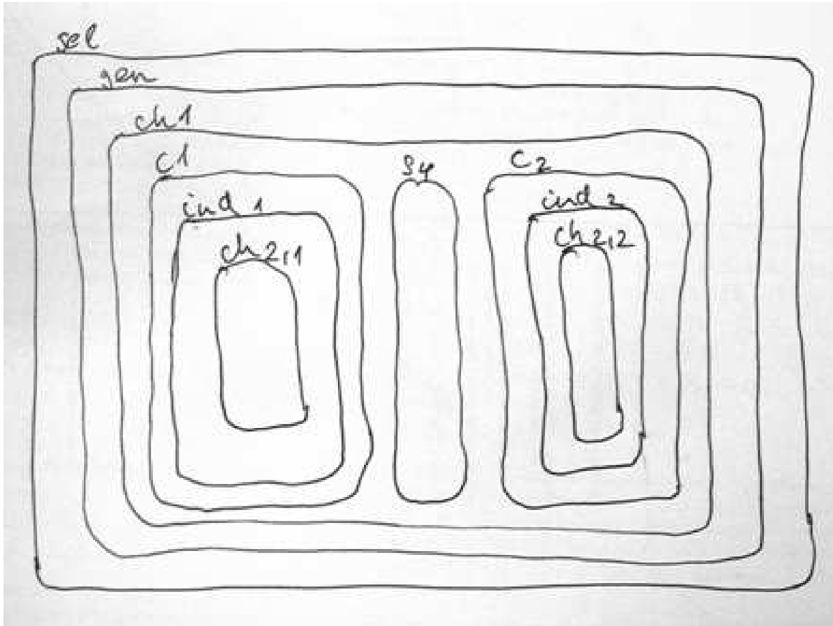
Region sel is for producing nonterminals

$$[q, a, c_1, c_2;\ q', e_1, e_2]$$

that correspond to the TCM transitions

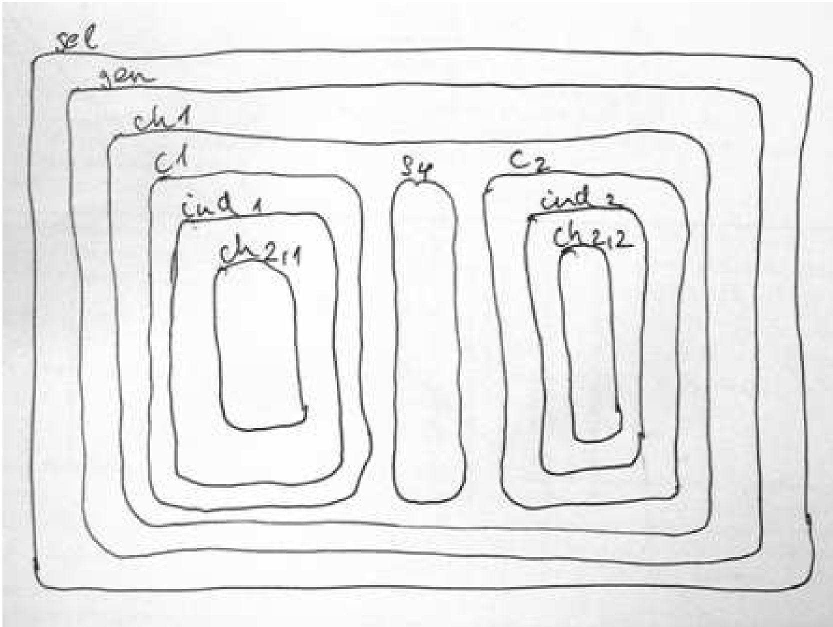
$$(q, a, c_1, c_2, ;\ q', e_1, e_2).$$

In the final phase, it also checks if the computation was correct.



The proof of $iRPC_{10}CF = RE$ continued

Region gen is responsible for **adding the symbol read by the TCM** to the word generated by the membrane system.
The generation of the word letter-by letter corresponds to the computation (reading) of the word by the TCM.

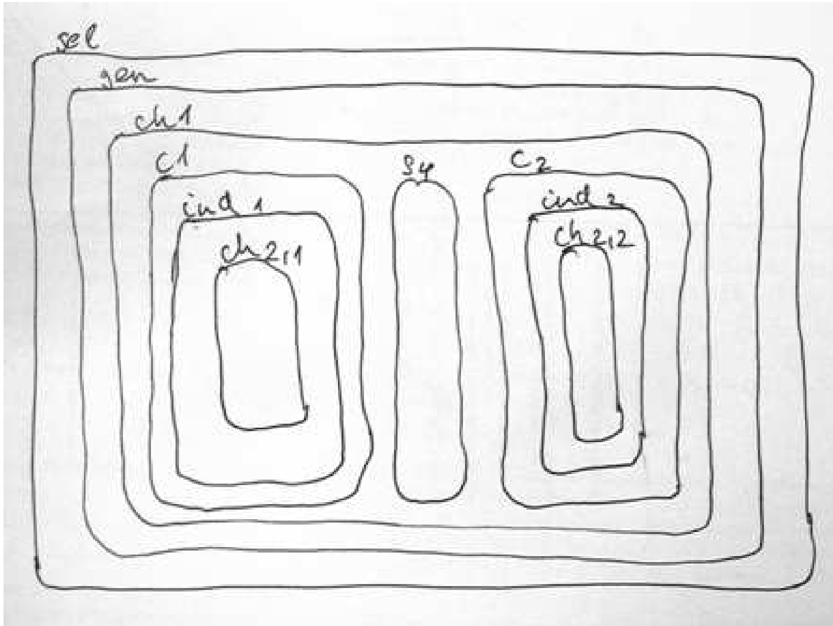


The proof of $iRPC_{10}CF = RE$ continued

Regions c_i simulate the changes in the contents of counters c_i. Strings **containing** $A \in N$ in the regions c_i

$$\underbrace{AA \ldots A}_{c_i}$$

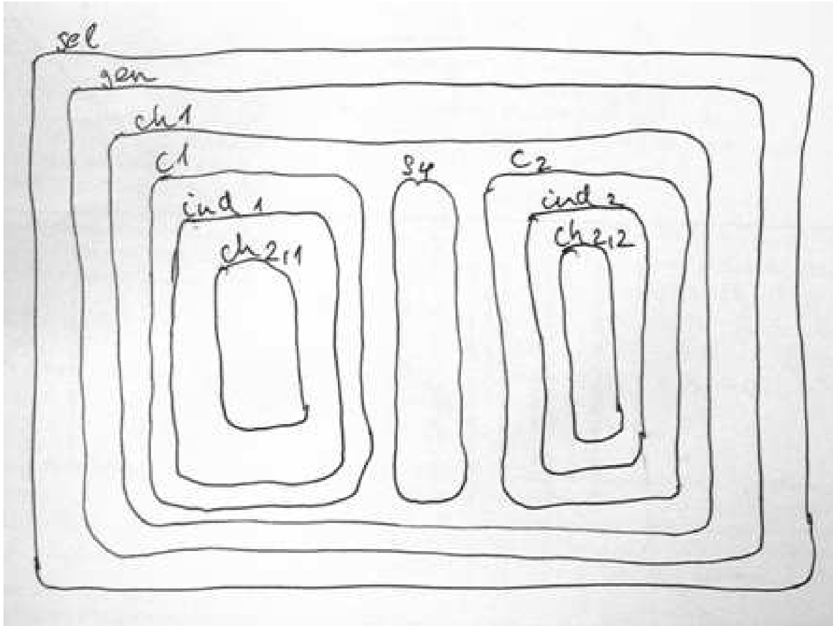
correspond to the **counter values** c_i.



The proof of $iRPC_{10}CF = RE$ continued

Region $ch1$ assists in **checking the correctness** of the simulation.

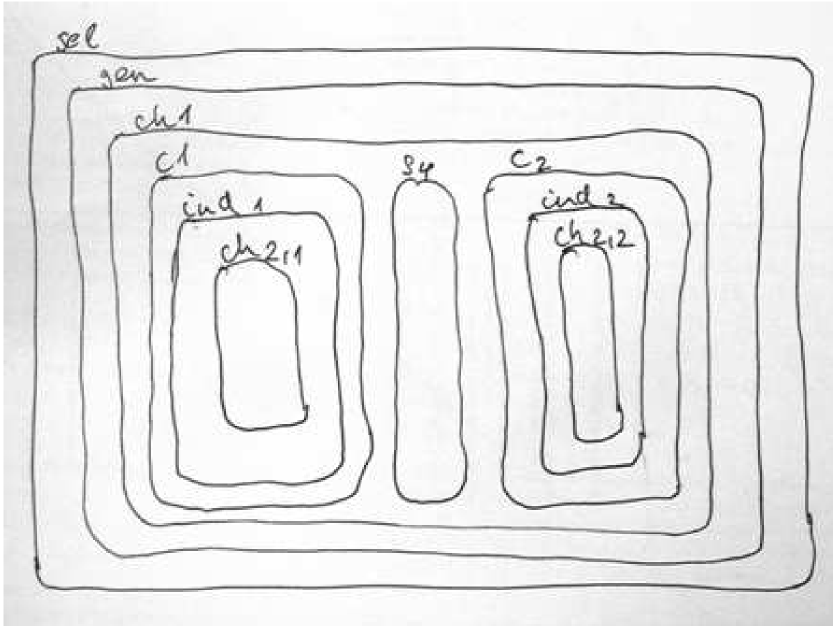
If **symbol** A **appears** in a word in region $ch1$, then an **instruction** of the TCM to be executed with an **empty counter** was not correctly simulated.



The proof of $iRPC_{10}CF = RE$ continued

Regions ind_i and $ch_{2,i}$ assists in **checking the correctness** of the simulation.

These regions guarantee that the simulation of the **decrement instructions** of the TCM can be performed iff the corresponding word in region c_i **contains at least one symbol** A.



Simulating a transition: $\alpha = [q, x, Z, B; q', +1, -1]$, $\delta_\alpha = AA$

	C_{sel}	C_{gen}	C_{ch_1}	C_{c_1}	C_{ind}	$C_{ch_{2,1}}$
0	β_8	wS_1, S'_1	$E...S_1, S'_1$	$E...J, C_1$	B_1	$AEJ...H_1$
1	α_1	wS_2, Q_{sel}	$E...S_2, S'_2$	$E...J_1, C_2$	B_2	$AEJ...H_2$
1C	α_1	wS_2, α_1	$E...S_2, S'_2$	$E...J_1, C_2$	B_2	$AEJ...H_2$
2	α_2	wQ_{sel}, α'_1	$E...S_3, Q_{gen}$	$E...J_2, C_3$	B_3	$AEJ...H_3$
2C	α_2	$w\alpha_2, \alpha'_1$	$E...S_3, \alpha'_1$	$E...J_2, C_3$	B_3	$AEJ...H_3$
3	α_3	wQ_{sel}, α''_1	$E...Q_{gen}, \bar{\alpha}'_1$	$E...Q_{ch_1}, C_4$	B_4	$AEJ...H_4$
3C	α_3	$w\alpha_3, \alpha''_1$	$E...\alpha''_1, \bar{\alpha}'_1$	$E...\bar{\alpha}'_1, C_4$	B_4	$AEJ...H_4$
4	α_4	$w\alpha_4, S'_2$	$E...Q_{c_1}Q_{S_4}, \underline{\bar{\alpha}}'_1$	$E...\bar{\alpha}''_1, Q_{ch_1}$	B_5	$AEJ...H_5$
4C	α_4	$w\alpha_4, S'_2$	$E...\bar{\alpha}''_1 S_4, \underline{\bar{\alpha}}'_1$	$E...\bar{\alpha}''_1, \underline{\bar{\alpha}}'_1$	B_5	$AEJ...H_5$
5	α_5	$w\alpha_5, S'_3$	$E...\bar{\alpha}''_1 S_5, S'_3$	$E...\delta_\alpha J_3, C_5$	B_6	$AEJ...H_6$
6	α_6	$w\alpha_6, S'_4$	$E...\bar{\alpha}''_1 S_6, S'_4$	$E...Q_{ind}J_3, C_6$	E	$AEJ...H_7$
6C	α_6	$w\alpha_6, S'_4$	$E...\bar{\alpha}''_1 S_6, S'_4$	$E...EJ_3, C_6$	E	$AEJ...H_7$
7	α_7	$w\alpha_7, S'_5$	$E...\bar{\alpha}''_1 S_7, S'_5$	$E...EQ_{ind}, C_7$	J_4	$AEJ...H_8$
7C	α_7	$w\alpha_7, S'_5$	$E...\bar{\alpha}''_1 S_7, S'_5$	$E...EJ_4, C_7$	J_4	$AEJ...H_8$
8	α_8	wxS_1, S'_1	$E...\bar{\alpha}''_1 S_1, S'_1$	$E...EJ, C_1$	B_1	$AEJQ_{c_1}H_1$
8C	α_8	wxS_1, S'_1	$E...\bar{\alpha}''_1 S_1, S'_1$	$E...EJ, C_1$	B_1	$AEJ...H_1$

Simulating the termination, moving the result w to C_{sel}.

	C_{sel}	C_{gen}	C_{ch_1}	C_{c_1}	C_{ind}	$C_{ch_{2,1}}$
0	β_8	wS_1, S'_1	$E...S_1, S'_1$	$E...J, C_1$	B_1	$AEJ...H_1$
1	F_1	wS_2, Q_{sel}	$E...S_2, S'_2$	$E...J_1, C_2$	B_2	$AEJ...H_2$
1C	F_1	wS_2, F_1	$E...S_2, S'_2$	$E...J_1, C_2$	B_2	$AEJ...H_2$
2	F_2	$wQ_{sel}, F_{1,1}$	$E...S_3, Q_{gen}$	$E...J_2, C_3$	B_3	$AEJ...H_3$
2C	F_2	$wF_2, F_{1,1}$	$E...S_3, F_{1,1}$	$E...J_2, C_3$	B_3	$AEJ...H_3$
3	F_3	$wQ_{sel}, F_{1,2}$	$E...Q_{gen}, \bar{F}'_1$	$E...Q_{ch_1}, C_4$	B_4	$AEJ...H_4$
3C	F_3	$wF_3, F_{1,2}$	$E...F_{1,2}, \bar{F}'_1$	$E...\bar{F}'_1, C_4$	B_4	$AEJ...H_4$
4	F_4	$wF_4, F_{1,3}$	$E...Q_{S_4}, \underline{\bar{F}'}_1$	$E...\bar{F}''_1, Q_{ch_1}$	B_5	$AEJ...H_5$
4C	F_4	$wF_4, F_{1,3}$	$E...S_4, \underline{\bar{F}'}_1$	$E...\bar{F}''_1, \underline{\bar{F}'}_1$	B_5	$AEJ...H_5$
5	F_5	$wF_5, F_{1,4}$	$E...S_5, \underline{\underline{\bar{F}'}}_1$	$E...\bar{F}''_1, Q_{ch_{2,1}}$	B_6	$AEJ...H_6$
5C	F_5	$wF_5, F_{1,4}$	$E...S_5, \underline{\underline{\bar{F}'}}_1$	$E...\bar{F}''_1, AEJ...H_6$	B_6	$AEJ...H_6$
6	F_6	wQ_{ch_1} $F_{1,5}$	$E...S_6$ $Q_{c_1}Q_{c_2}$	$E...\bar{F}''_1$ $AEJ...H_7$	E	$AEJ...H_7$
6C	F_6	$wE...S_6$ $F_{1,5}$	$E...S_6$ $AEJ......H_7$	$E...\bar{F}''_1$ $AEJ...H_7$	E E	$AEJ...H_7$ $AEJ...H_7$
7	Q_{gen}	$wE...S'_6$ Q_{ch_1}	$E...S_7$ $AEJ......H_7$	$E...\bar{F}''_1$ $AEJ...H_7$	J_4	$AEJ...H_8$
7C	$wE...S'_6$	$wE...S'_6$ $AEJ...H_7$	$E...S_7$ $AEJ......H_7$	$E...\bar{F}''_1$ $AEJ...H_7$	J_4	$AEJ...H_8$

Thus, combining our results, we obtain

$$\boxed{iRPC_{10}CF = pRPC_{30}CF = RE}$$

Conclusions

- RPC systems are as powerful as the Turing machines

- The two variants of communication do not imply difference in the generative power

- The organization of the membrane structure (standard, tissue-like) does not imply difference in the generative power

Open problems

- How to model biological phenomena as resistance, malignant behaviour, etc. in this framework?

- These systems can be considered as models of assembly as well. How they can be related to the existing models?

- What about the communication complexity of these constructs?