

# A cellular solution to Subset Sum using non-elementary division and dissolution, with time and initial resources bounded by $\log k$

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  - Formal Framework
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  - An overview of the computations
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# P systems with active membranes

with weak non-elementary division and without polarizations

## Formal Definition

- $\Gamma$  (working alphabet)
- $H$  (labels for membranes)
- $\mu$  (membrane structure)
- $M$  (initial multisets)
- Rules:
  - (a)  $[a \rightarrow u]_h$
  - (b)  $a [ ]_h \rightarrow [b]_h$
  - (c)  $[a]_h \rightarrow b [ ]_h$
  - (d)  $[a]_h \rightarrow b$
  - (e)  $[a]_h \rightarrow [b]_h [c]_h$

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# Recognizing P systems

- Decision problems.
- First receive an input (via a **multiset**).
- Creation of an **exponential** number of membranes.
- **Parallel** evolution.
- Final stage check/answer

## Recognizing P systems (cont.)

### Definition

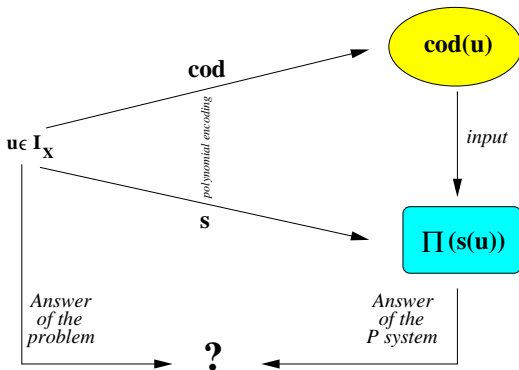
A *recognizing P system* is a P system with input membrane and external output such that:

- *Yes*, *No*  $\in \Gamma$  (working alphabet).
- all the computations *halt*.
- for every computation, one symbol *Yes* or one symbol *No* (*but not both*) is sent out (and in the *last step* of the computation).

$AM^0(+d, +ne)$ : class of *polarizationless P systems using weak division of non-elementary membranes and dissolution*.

# Uniform designs: sketch

$$X = (I_X, \theta_X) \rightsquigarrow F_X = (\Pi(n))_{n \in \mathbb{N}}$$





# Computational complexity framework (informal)

M.J. Pérez, A. Romero, F. Sancho, 2002

## Definition

Let  $\mathcal{R}$  be a class of recognizing P systems. A decision problem  $X$  is **solvable in polynomial time** by a family  $F_X$ , of  $\mathcal{R}$ , if

- elements of  $F_X$  can be built **polynomially**.

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- elements of  $F_X$  can be built polynomially.
- There exists a pair  $(cod, s)$  of pol-time computable functions such that
  - elements of  $F_X$  run **polynomially**, with regard to  $(X, cod, s)$ .
  - $F_X$  is **sound** and **complete**, with regard to  $(X, cod, s)$ .

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  - $F_X$  is sound and complete, with regard to  $(X, cod, s)$ .

We denote this by  $X \in \mathbf{PMC}_{\mathcal{R}}$ .

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# Solving the Subset Sum Problem

## Subset Sum problem

Given a finite set  $A$ , a weight function  $w : A \rightarrow \mathbb{N}$ , and a constant  $k \in \mathbb{N}$ , determine whether or not there exists a subset  $B \subseteq A$  such that  $w(B) = k$ .

## Stages of the solution

- *Preparation*:  $\log k$  membranes  $ch \Rightarrow k$  membranes.
- *Generation*: obtain by division  $2^n$  membranes  $e$  (non-elem).
- *Weight calculation*: for each possible subset.  
WAIT / SYNCHRO
- *Checking*:  $w(B) = k?$   
WAIT / SYNCHRO
- *Output*: send either yes or no.

# A small example: $n = 3, k = 5$

$$A = \{a_1, a_2, a_3\}, w(a_1) = 3, w(a_2) = 1, w(a_3) = 6$$

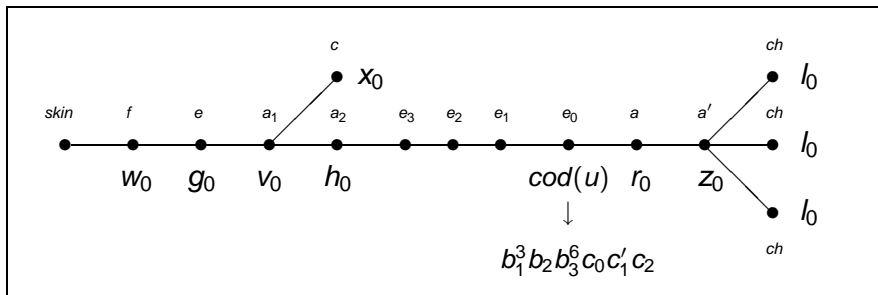


Figure: Initial Configuration

$$A = \{a_1, a_2, a_3\}, w(a_1) = 3, w(a_2) = 1, w(a_3) = 6$$

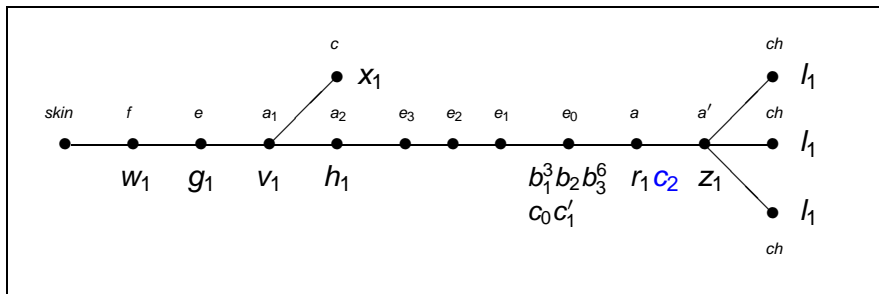


Figure: *Time = 1*



$$A = \{a_1, a_2, a_3\}, w(a_1) = 3, w(a_2) = 1, w(a_3) = 6$$

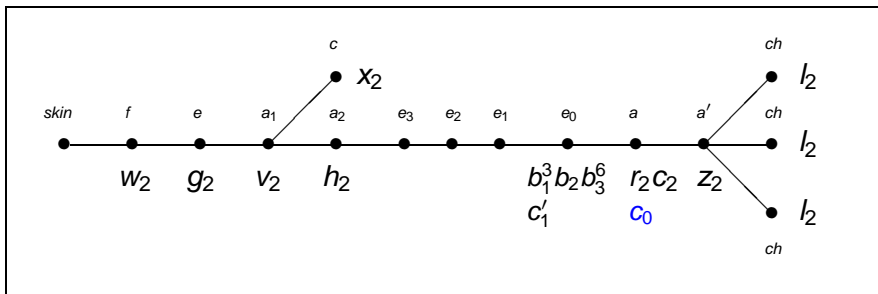


Figure: *Time = 2*

$$A = \{a_1, a_2, a_3\}, w(a_1) = 3, w(a_2) = 1, w(a_3) = 6$$

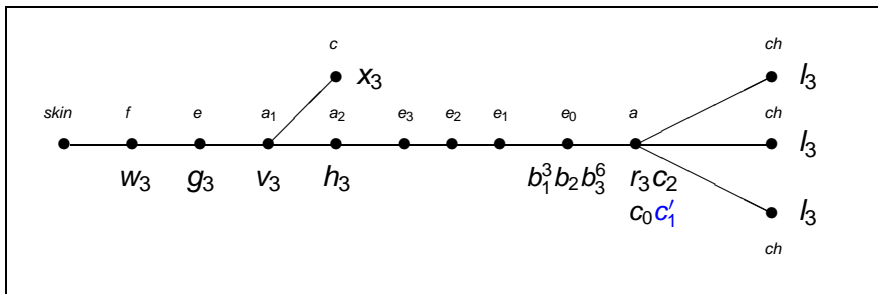


Figure:  $Time = 3$  ( $z_2$  dissolves membrane  $a'$ )

$$A = \{a_1, a_2, a_3\}, w(a_1) = 3, w(a_2) = 1, w(a_3) = 6$$

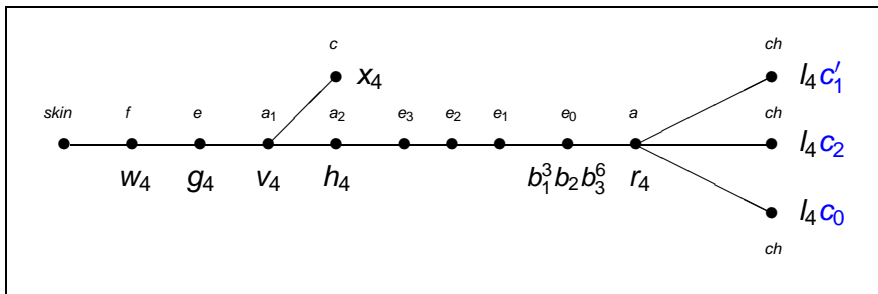


Figure: *Time = 4*

$$A = \{a_1, a_2, a_3\}, w(a_1) = 3, w(a_2) = 1, w(a_3) = 6$$

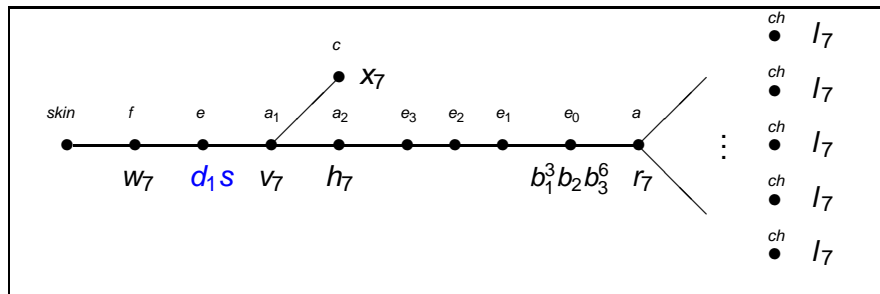


Figure:  $Time = 7 = 2\lfloor \log k \rfloor + 3$

# Final remarks

## Looking for Computational Power / Efficiency

Can objects be “aware” of the situation in their region?

- Context sensitivity
- Changing membrane charge / label
- Dissolution rules (irreversible)

## Future work

- Decreasing number of counters
- Removing / restricting other ingredients
- ...

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THANK YOU!