Frequency Membrane Systems

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Timing and Synchronization – literature

- Parameters considered in P-System variants include:
  - Maximal parallelism
  - Non-Determinism
  - Existence of a global clock
  - Duration of rules' application
  - *Decay time of symbols*
- Other features
  - Energy in cells / Energy varied by rules
    (see also UREM P-systems [Freund et al., 2004])

Frequency Membrane Systems

- We mainly wanted to introduce richer patterns on timing:
- There is a global clock, but...
- ...each membrane can work with periods of length multiple of the length of a global clock's period

(or, conversely, they have a frequency which is a fraction of the global clock's frequency)

- Each local clock could start with an offset w.r.t. the global clock

Details of Frequency Membrane Systems

- Parameters changed in this P-System variant include:
  - In each membrane, rules to be applied are selected sequentially at each step (Not Maximal parallelism)
  - This expects Non-Determinism
  - Existence of a global clock, PLUS local clocks
  - Duration of rules' application can be >=0 ticks
  - *Decay time of symbols*: can be a finite time
- Other features
  - Energy in cells: set at each step / Energy consumed by rules

A Frequency P system with symbol objects of degree $m \geq 1$, is a construct

$$\Pi = (O, D, T, \mu, \omega_1, \ldots, \omega_m, E, t_D, C, R_1, \ldots, R_m, i_O)$$

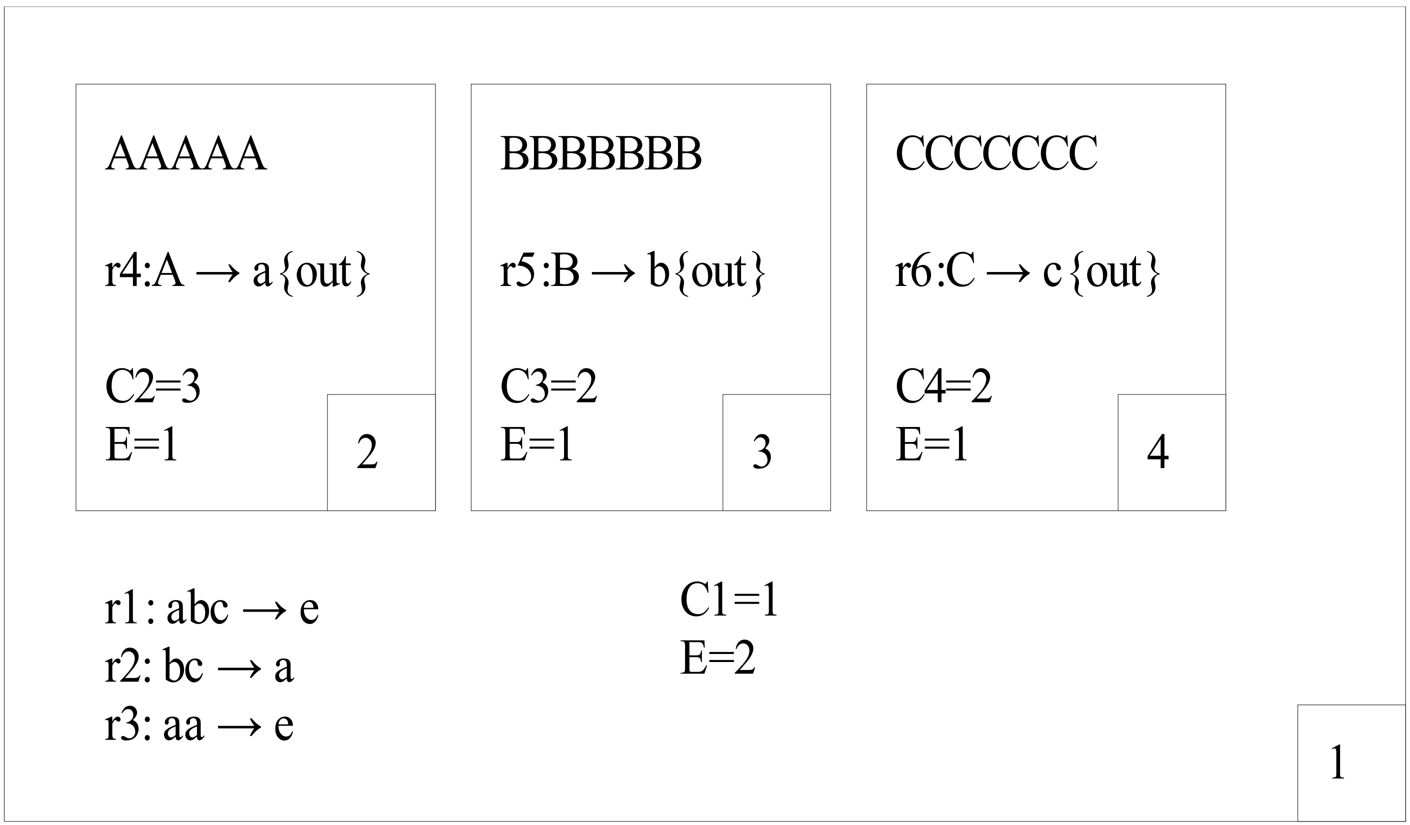
where:

- O is the alphabet of the objects;
- $D \subseteq O$ is the alphabet of decaying symbols;
- $T \subseteq O$ is the alphabet of non-decaying symbols;
- μ is a membrane structure consisting of m membranes labeled with 1,2,. . . ,m;
- $\omega_i, 1 \leq i \leq m$, specifies the multiset of objects present in the corresponding region i at the beginning of a computation;
- $E \subseteq N^m$ is a set of m numbers indicating the energy value assigned to each membrane at every membrane's clock step, overriding any previous energy level associated to them;
- $t_D \subseteq N^n$ is a set of n numbers indicating the decay time of the n decay symbols in D;
- $C \subseteq N^m$ is a set of m numbers indicating the clock value (referred to an external observer) assigned on each membrane;
- $R_i, 1 \leq i \leq m$, are finite sets of evolutionary rules over O associated with regions 1,2,. . . ,m of μ; the rules can be either cooperative or non-cooperative rules of the form $s \longrightarrow^k_s c$, where s is a string over O c is over $\{a_{here}, a_{out}, a_j \mid a \text{ in } O, 1 \leq j \leq m\}$, if the target is not specified, then it is intended to be *here*; k is an integer representing the energy to consume to apply the rule. Note that k could be a negative number, in this case we assume that the reaction modeled by the rule produces energy for the cell, when k is not specified we assume that k=1; s in N is the number of clock's steps necessary to the rules to act (and produce the objects in the right hand side), when s is not specified we assume that s=0;
- i_O in {0,1,. . . ,m} is the output region (0 for the environment).

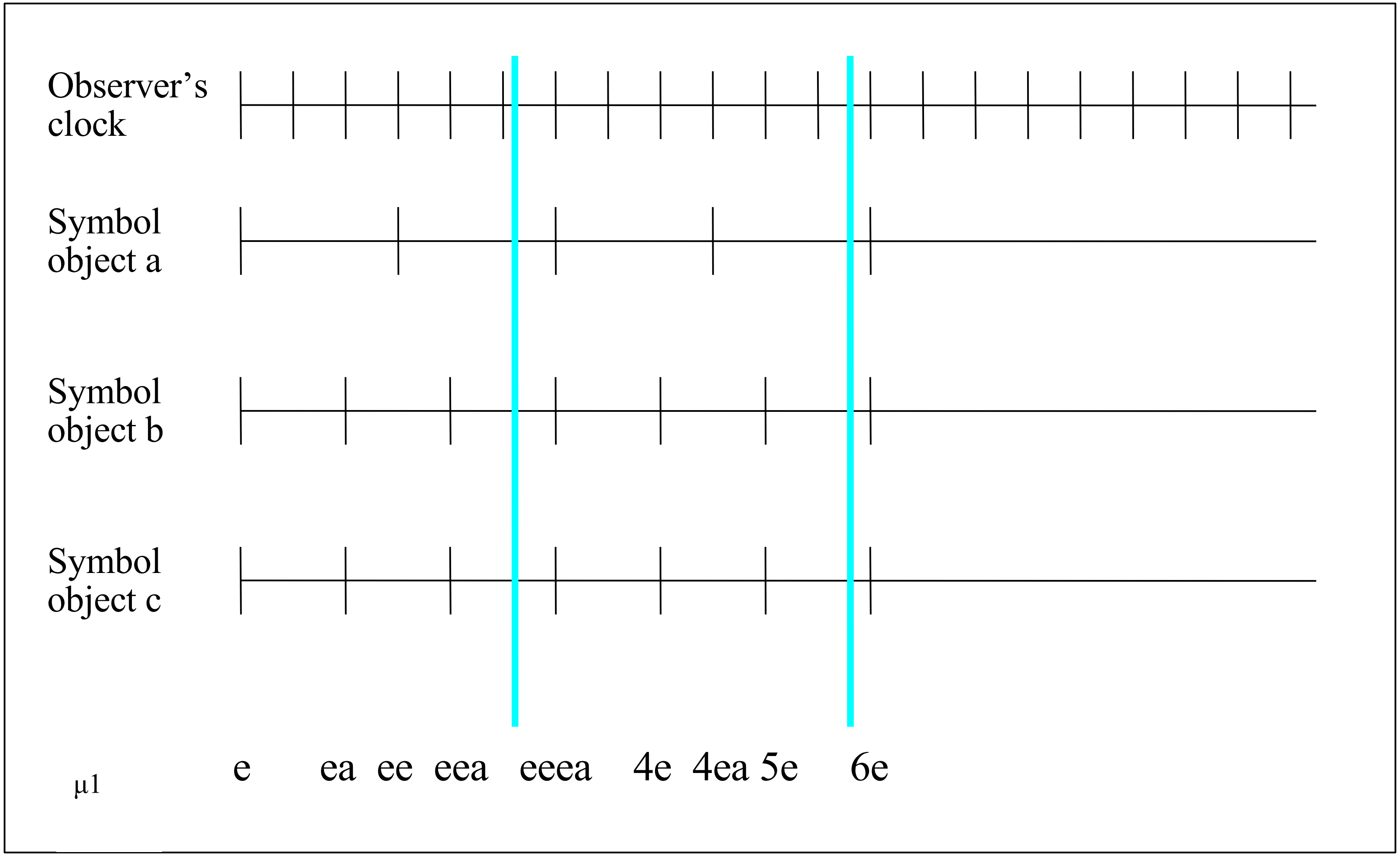
Results and Goals

- Frequency P-Systems are an extension of the original:
  - Clocks can be identical, decay and energy can be infinite, ...
- It's easy to show that the decay time of symbols is the most disruptive parameter
- We look for:
  - Universal computation (RE, ...)
  - Dynamics like: oscillation, self-synchronization among membranes
  - Systems which do not depend on local offsets, ND-choices, ...

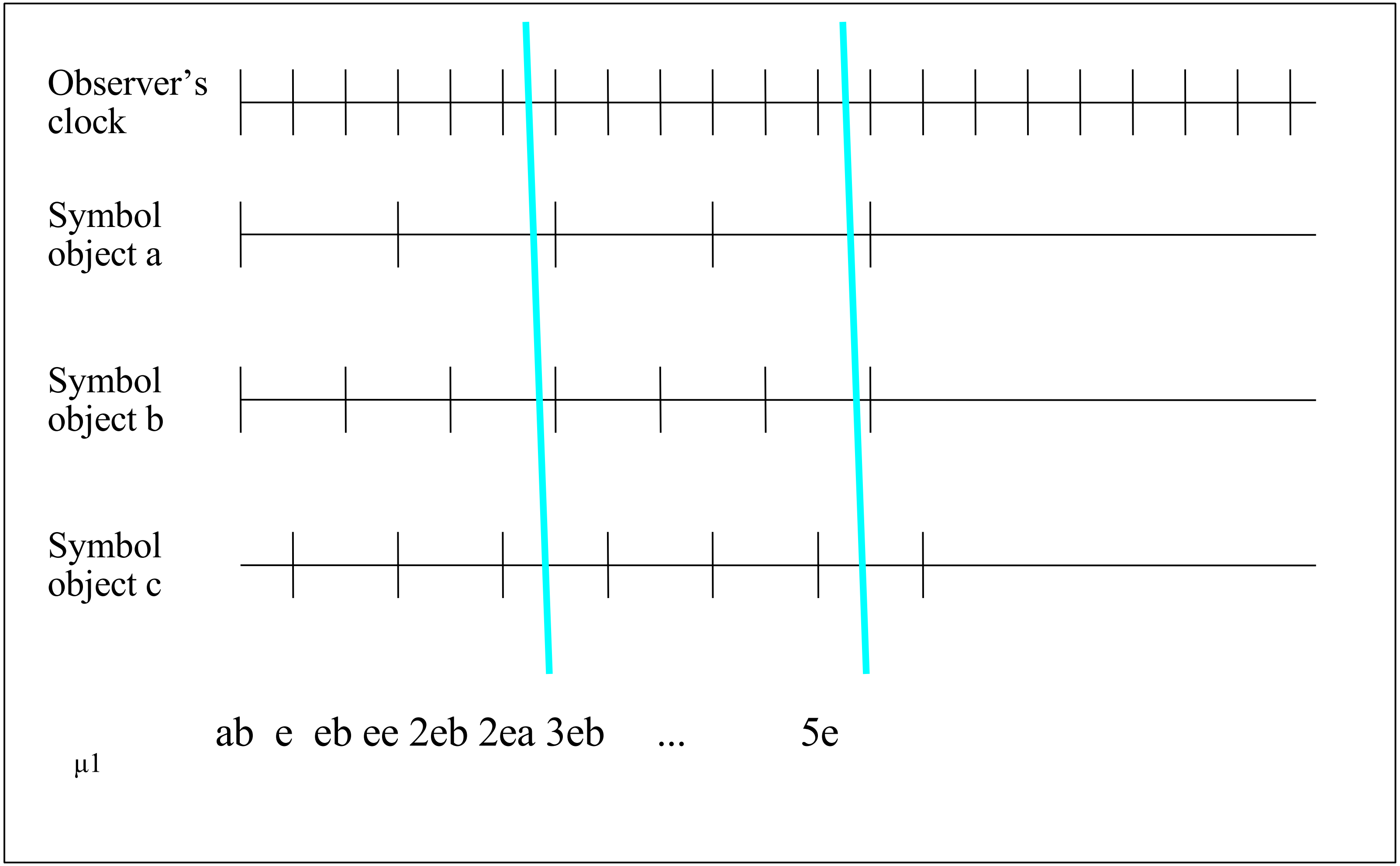
An example


AAAAA
r4:A → a{out}
C2=3
E=1
2
BBBBBBB
r5:B → b{out}
C3=2
E=1
3
CCCCCCC
r6:C → c{out}
C4=2
E=1
4
r1: abc → e
r2: bc → a
r3: aa → e
C1=1
E=2
1

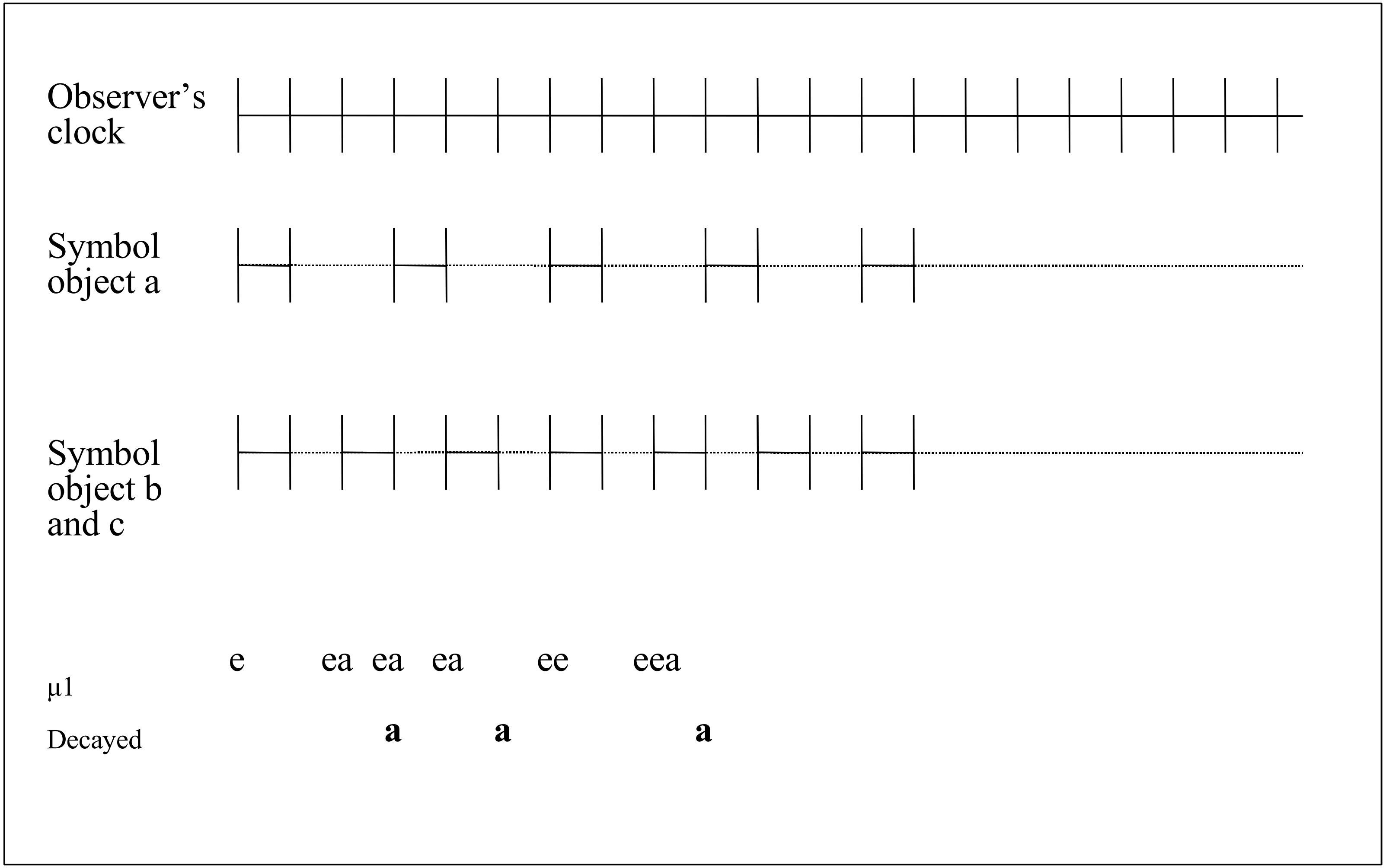
An example – dynamics without decay



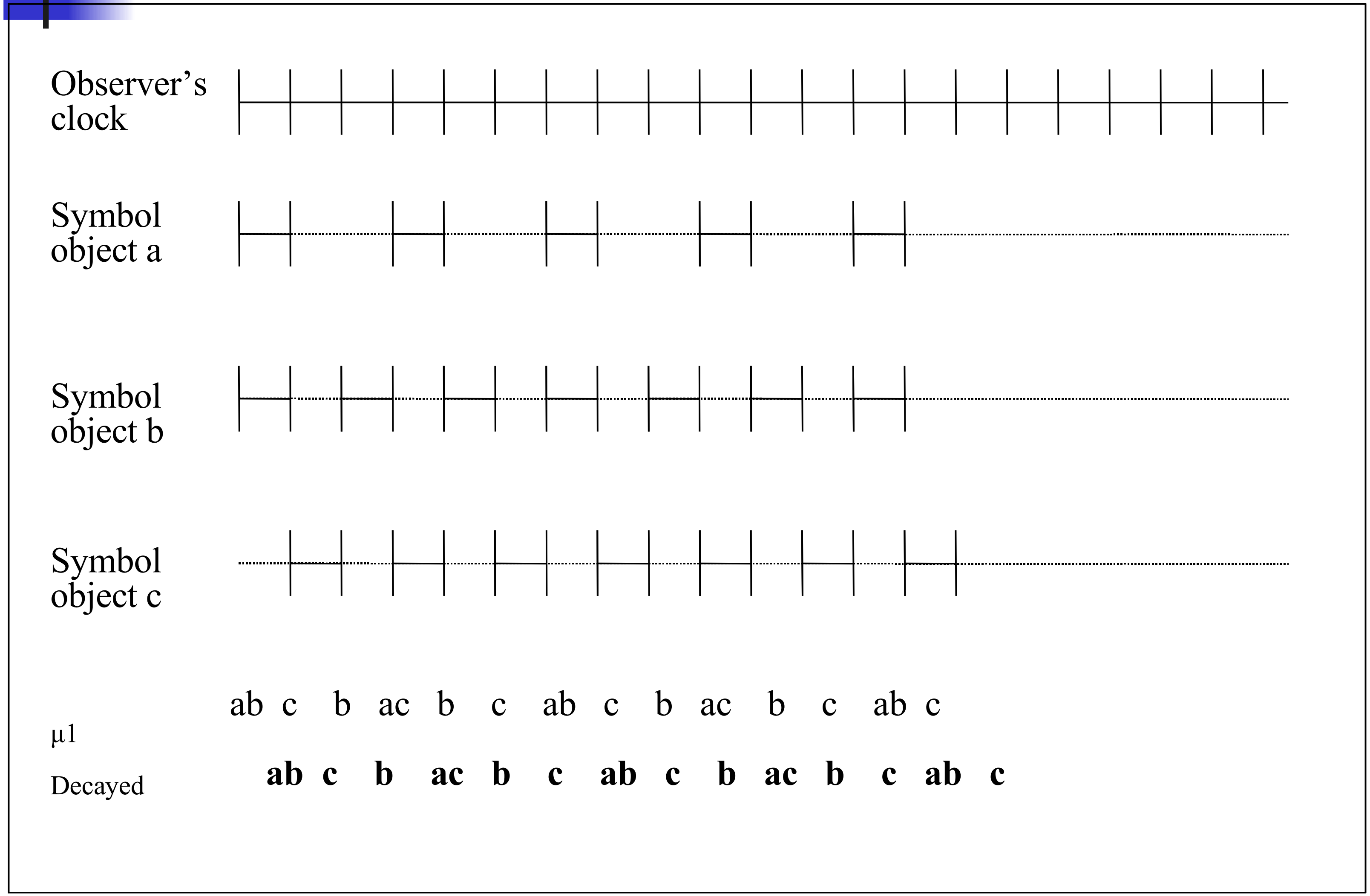
An example–offset dynamics without decay



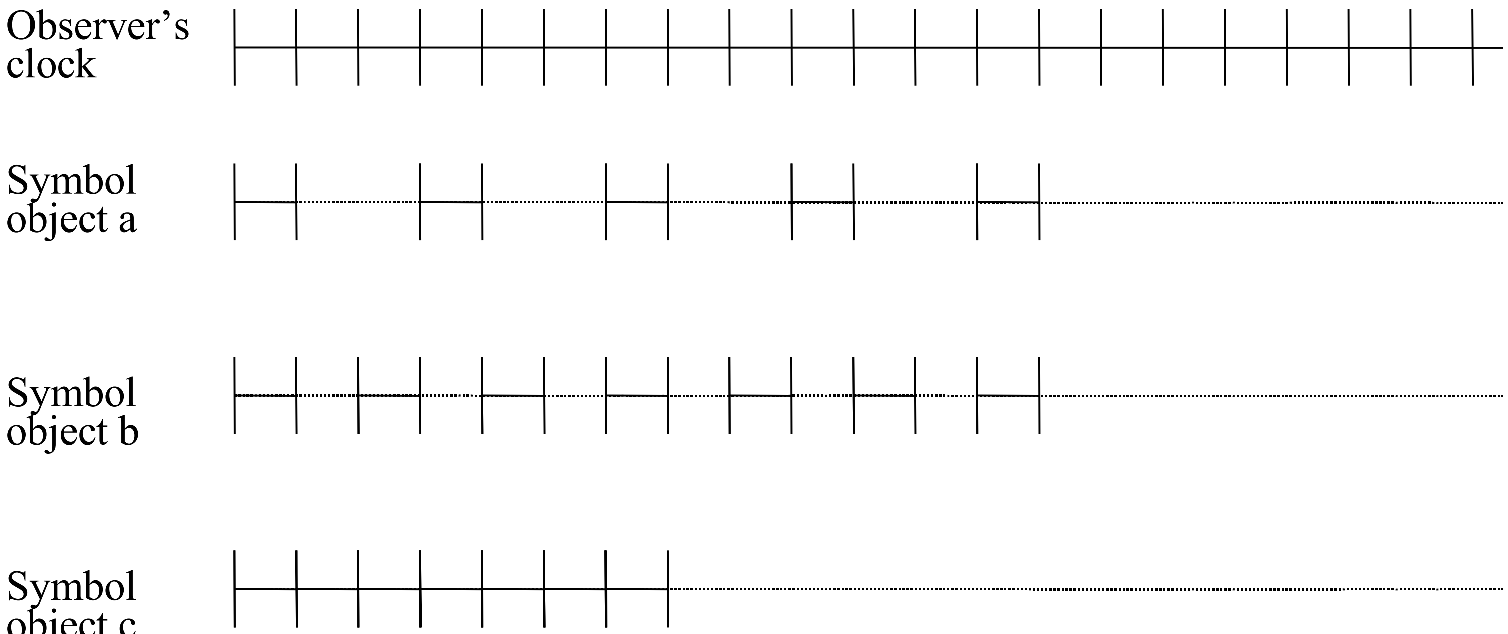
An example – dynamics with decay=1



An example –offset dynamics with decay=1



Higher frequency – dynamics with decay=1



μ1 e ec ea eac ea ec ee ….

Decayed **c a ac a c …**

A result about comparison to P-systems

- Frequency P systems not consuming energy and without decaying symbols can be simulated by usual P systems with maximal parallelism:
- in each membrane there are “ticking objects and rules”, for a simulated membrane with period length of 3:

  t0 -> t1 ->t2 ->t0

- each simulated rule u->v becomes (cooperative):

  t0 u -> t? v

Simulating timed rules

- If simulated rule takes 0 steps: t0u -> v
- Simulated rule takes 2 steps: t0u -> prj(t2,v), t2<t2,c> ->c

  where ... prj(t2,c)=<t2,c>, with <t2,c> new symbol

  (t0u -> t2 v would be wrong) (number of steps < period!)

Further remarks:

- membranes start with as many t0 objects as the number of simulated starting objects, then specific rules multiply them (not efficient)
- to impose a starting offset of 2 to a membrane it suffices to start with t(period - 2) symbols, instead of t0 (limit to delay)

Further extensions

- Timing could be dependent on membrane contents (as studied by Cavaliere et al.)
- This is similar to our decay time associated to some symbols, which can be used as timing signals, which can change during the computation