On the Reachability Problem in P Systems with Mobile Membranes

Bogdan Aman[1] Gabriel Ciobanu[1,2]

[1]Romanian Academy, Institute of Computer Science
Blvd. Carol I no.8, 700505 Iași, Romania
[2]"A.I.Cuza" University of Iași, Faculty of Computer Science
Blvd. Carol I no.11, 700506 Iași, Romania
baman@iit.tuiasi.ro, gabriel@info.uaic.ro

8th Workshop on Membrane Computing
Thessaloniki, Greece,
25 - 28 June, 2007

Outline

Mobile Membrane Systems

Definition

A mobile membrane system is a construct

$$\prod = (V \cup \overline{V}, H \cup \overline{H}, \mu, w_1, \ldots, w_n, R), \quad \text{where:}$$

1. $n \geq 1$ (the initial degree of the system);
2. $V \cup \overline{V}$ is an alphabet (its elements are called objects), where $V \cap \overline{V} = \emptyset$;
3. $H \cup \overline{H}$ is a finite set of labels for membranes, where $H \cap \overline{H} = \emptyset$;
4. μ is a membrane structure, consisting of n membranes, labelled (not necessarily in a one-to-one manner) with elements of H;
5. $w_1, w_2, \ldots, w_n$ are multisets of objects placed in the n membranes of the system;
6. R is a finite set of developmental rules, of the following forms:

Mobile Membrane Systems

(a) $\overline{a}\downarrow \rightarrow \overline{a}\downarrow\ a\downarrow$, for $a\downarrow \in V$, $\overline{a}\downarrow \in \overline{V}$; replication rule
The objects $\overline{a}\downarrow$ are used to create new objects $a\downarrow$ without being consumed.

(b) $\overline{a}\uparrow \rightarrow \overline{a}\uparrow\ a\uparrow$, for $a\uparrow \in V$, $\overline{a}\uparrow \in \overline{V}$; replication rule
The objects $\overline{a}\uparrow$ are used to create new objects $a\uparrow$ without being consumed.

(c) $[\ a\downarrow\ ]_h\ [\ ]_a \rightarrow [\ [\ ]_h\ ]_a$, for $a, h \in H, a\downarrow \in V$; endocytosis
An elementary membrane labelled h enters the adjacent membrane labelled a, under the control of object $a\downarrow$. The labels h and a remain unchanged during this process; however the object $a\downarrow$ is consumed during the operation. Membrane a is not necessarily elementary.

(d) $[\ [\ a\uparrow\ ]_h\ ]_a \rightarrow [\ ]_h\ [\ ]_a$, for $a, h \in H, a\uparrow \in V$; exocytosis
An elementary membrane labelled h is sent out of a membrane labelled a, under the control of object $a\uparrow$. The labels of the two membranes remain unchanged; the object $a\uparrow$ of membrane h is consumed during the operation. Membrane a is not necessarily elementary.

(e) $[\ ]_{\overline{h}} \rightarrow [\ ]_{\overline{h}}[\ ]_h$ for $h \in H$, $\overline{h} \in \overline{H}$ division rules
An elementary membrane labelled $\overline{h}$ is divided into two membranes labelled by $\overline{h}$ and h having the same objects.

Mobile Ambients

Syntax

$$C \quad ::= \quad in\ n \quad | \quad out\ n \qquad \textbf{Capabilities}$$
$$A \quad ::= \quad A \mid B \quad | \quad C\,.\,A \quad | \quad n[\,A\,] \quad | \quad !A \qquad \textbf{Processes}$$

Axioms

(In) $n[\ in\ m.\,A \mid A'\ ] \mid m[\ B\ ] \ \Rightarrow\ m[\ n[\ A \mid A'\ ] \mid B\ ]\ ;$

(Out) $m[\ n[\ out\ m.\,A \mid A'\ ] \mid B\ ] \ \Rightarrow\ n[\ A \mid A'\ ] \mid m[\ B\ ]\ ;$

(Repl) $!A \Rightarrow\ A \mid !A\ .$

Rules

(Comp) $\dfrac{A \ \Rightarrow\ A'}{A \mid B \ \Rightarrow\ A' \mid B}$ **(Amb)** $\dfrac{A \ \Rightarrow\ A'}{n[\ A\ ] \ \Rightarrow\ n[\ A'\ ]}$

(Struc) $\dfrac{A \equiv A',\ A' \ \Rightarrow\ B',\ B' \equiv B}{A \ \Rightarrow\ B}\ .$

Mobile Ambients

Structural Congruence

$A \,|\, B \equiv B \,|\, A$
$(A \,|\, B) \,|\, A' \equiv A \,|\, (B \,|\, A')$
$A \equiv A$
$A \equiv B$ implies $B \equiv A$
$A \equiv B$, $B \equiv A'$ implies $A \equiv A'$

$A \equiv B$ implies $A \,|\, A' \equiv B \,|\, A'$
$A \equiv B$ implies $!A \equiv !B$
$A \equiv B$ implies $n[A] \equiv n[B]$
$A \equiv B$ implies $C.A \equiv C.B$

Reachability Problem

Main Result

Theorem

For two arbitrary mobile membranes M_1 and M_2, it is decidable whether M_1 reduces to M_2.

Steps of the proof

1. we reduce mobile membranes systems to pure and public mobile ambients without the capability *open*.
2. we show that the reachability problem for two arbitrary mobile membranes can be expressed as the reachability problem for the corresponding mobile ambients.
3. we use the result that the reachability problem is decidable for a fragment of pure and public mobile ambients without the capability *open*.

From Mobile Membranes to Mobile Ambients

Step 1: mobile membranes are translated into pure public mobile ambients without capability *open*

Translation steps

1. any object $a\downarrow$ is translated into a capability *in a*;
2. any object $a\uparrow$ is translated into a capability *out a*;
3. any object $\overline{a}\downarrow$ is translated into a replication $!in\ a$
4. any object $\overline{a}\uparrow$ is translated into a replication $!out\ a$
5. a membrane h is translated into an ambient h
6. an elementary membrane $\overline{h}$ is translated into a replication $!h[\ ]$ where all the objects inside membrane h are translated into capabilities in ambient h using the above steps.

From Mobile Membranes to Mobile Ambients

Step 2: reachability problem for two arbitrary mobile membranes can be expressed as the reachability problem for the corresponding mobile ambients

Correspondence between rules

- rule (c) of mobile membranes corresponds to rule **(In)**
- rule (d) of mobile membranes corresponds corresponds to rule **(Out)**
- rules $(a), (b), (e)$ correspond to instances of rule **(Repl)**

Example

$M = [m{\downarrow}\ m{\uparrow}]_n[\ ]_m$ is translated into $\mathcal{T}(M) = n[in\ m \mid out\ m] \mid m[\ ]$.

Proposition

For mobile membrane systems M and N, M reduces to N by applying one rule if and only if $\mathcal{T}(M)$ reduces to $\mathcal{T}(N)$ by applying only one reduction rule.

From Mobile Ambients to Petri Nets

Labelled Transition System

In order to uniquely identify all the occurrences of replication, ambient, capability or hole $\Box$ within an ambient context or a process, we introduce a labelling system.

As an example, we give the labelled transition system associated with the process $n[!^1 in\ m.!^2 out\ m] \mid m[\ ]$ (we omit unnecessary labels). We use the labelled replications $!^1$ and $!^2$ to distinguish between different replication operators which appear in the process.

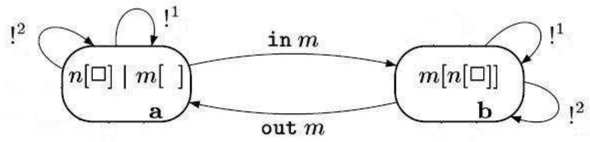

Fig. 1. A labeled transition system for the process $n[!^1 \mathbf{in}\ m.!^2 \mathbf{out}\ m] \mid m[\ ]$

As an example, for the process $!^1 in\ m.!^2 out\ m$, we obtain the Petri net given below:

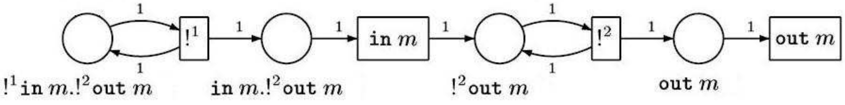

Fig. 2. A Petri net for the process $!^1\mathbf{in}\ m.!^2\mathbf{out}\ m$

From Mobile Ambients to Petri Nets

Combining the Transition System and Petri Nets

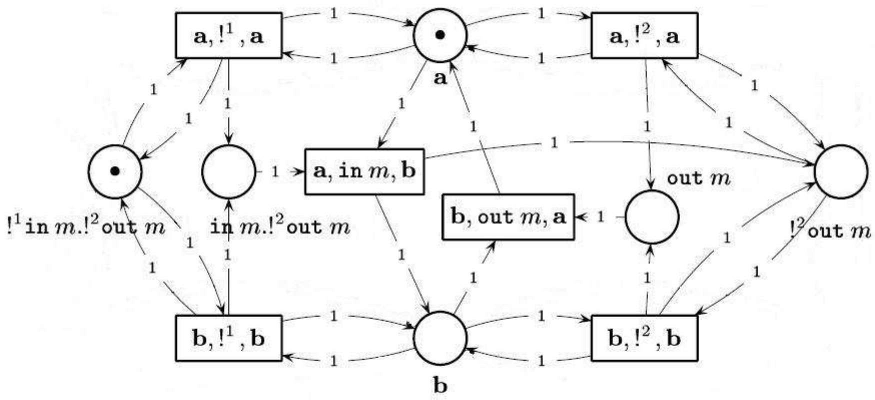

Fig. 3. The Petri net for the labeled process $n[!^1\,\mathbf{in}\ m.!^2\mathbf{out}\ m\ ]\ |\ m[\ ]$

Deciding Reachability

Step 3: reachability problem is decidable for a fragment of pure and public mobile ambients without the capability open.

Theorem (Mayr84)

For all Petri nets P, for all markings m, m' for P, one can decide whether m' is reachable from m.

Proposition: For a Petri net $PN_{A,B}$, there are only finitely many markings corresponding to the process B, and their set $\mathcal{M}_B$ can be computed.
Proposition: For all ambient processes A, B we have that $A \Rightarrow B$ if and only if there exists a marking from $\mathcal{M}_B$ such that m_B is reachable from m_A in $PN_{A,B}$.

Theorem

For two arbitrary ambients A and B from our restricted fragment, it is decidable whether A reduces to B.

Conclusions

- We have investigated the problem of reaching a configuration starting from another configuration in mobile membranes.

- We proved that the reachability can be decided by reducing this problem to the reachability problem of a version of pure and public ambient calculus without open capability and using a result of Boneva and Talbot.