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Characterizing Membrane Structures through Multiset Tree Automata

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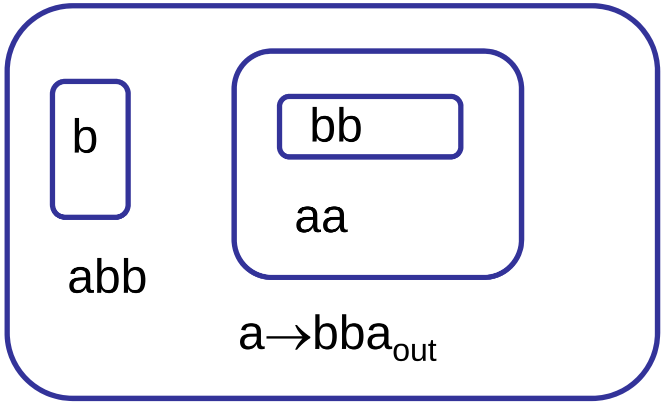
http://www.dsic.upv.es/users/tlcc/tlcc.html

Outline

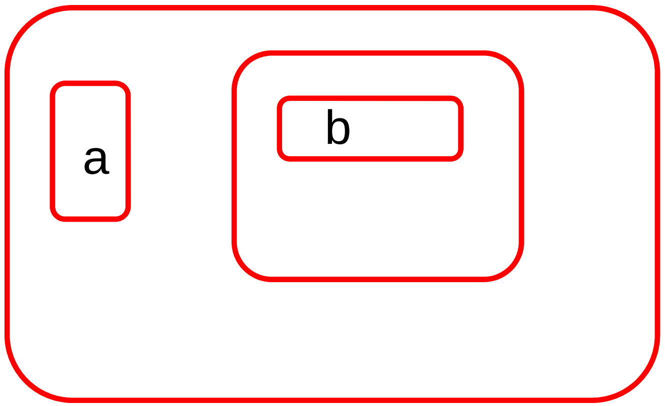
1 Basic concepts: Membrane structures and multiset tree automata
2 Basic concepts: *k*-Testability and *k*-reversability
3 Relations between *k*-Testability and *k*-reversability
4 From Multiset Tree Automata to Membrane Structures
5 Conclusions and future research

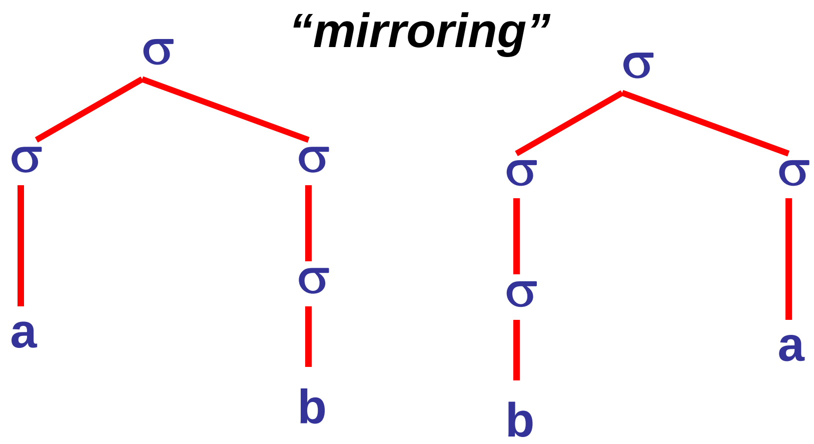
P systems and membrane structures

$\prod = (V, T, C, \mu, w_1, \ldots, w_m, (R_1,\rho_1), \ldots, (R_m\ \rho_m), i_0)$



$\prod = (V, T, C, \mu, w_1, \ldots, w_m, (R_1,\rho_1), \ldots, (R_m\ \rho_m), i_0)$





Multiset Tree Automata (MTA)

$\Pi = (Q, V, \delta, F)$

V is a *ranked alphabet* ($V' \times N$)

$V_i = \{ a \in V' : (a,i) \in V \}$

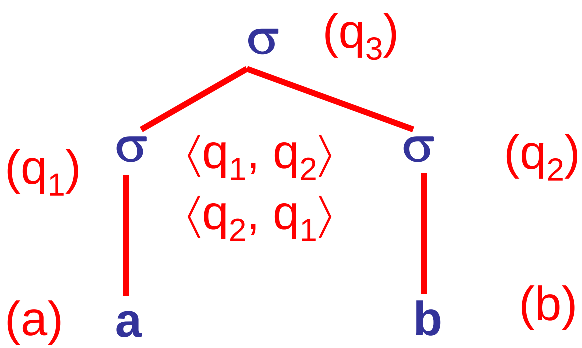
$\delta_i: V_i \times M_i\,(Q \cup V_0) \rightarrow P\,(M_1(Q))$

$M_i(\cdot)$ is a *bounded multiset* (the sum of elements is i)

$\delta_0(a) = M_\Psi(a) \in M_1\,(Q \cup V_0)$

$$\delta = \bigcup_{1 \leq i \leq maxarity(V)} \delta_i$$

Any multiset tree automaton performs a bottom-up parsing over any tree

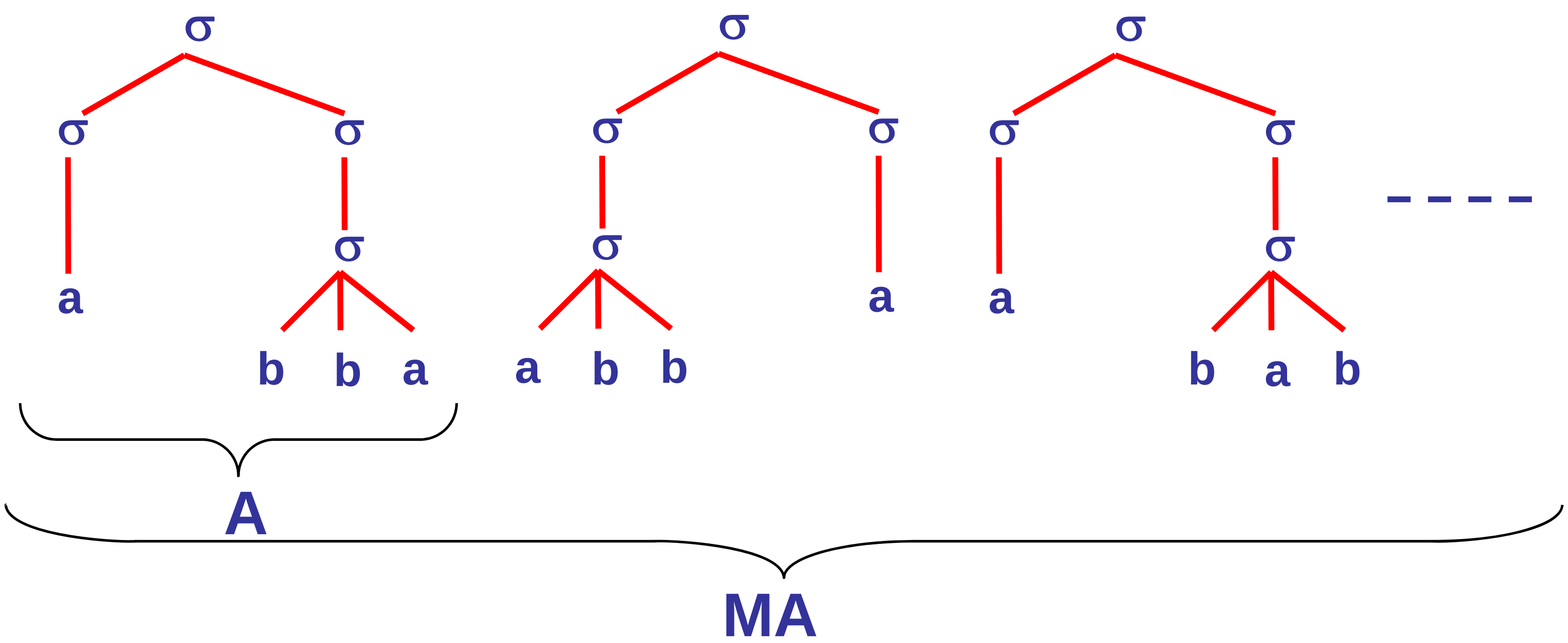


$\langle q_3 \rangle \in \delta_2(\sigma, \langle q_1, q_2 \rangle)$

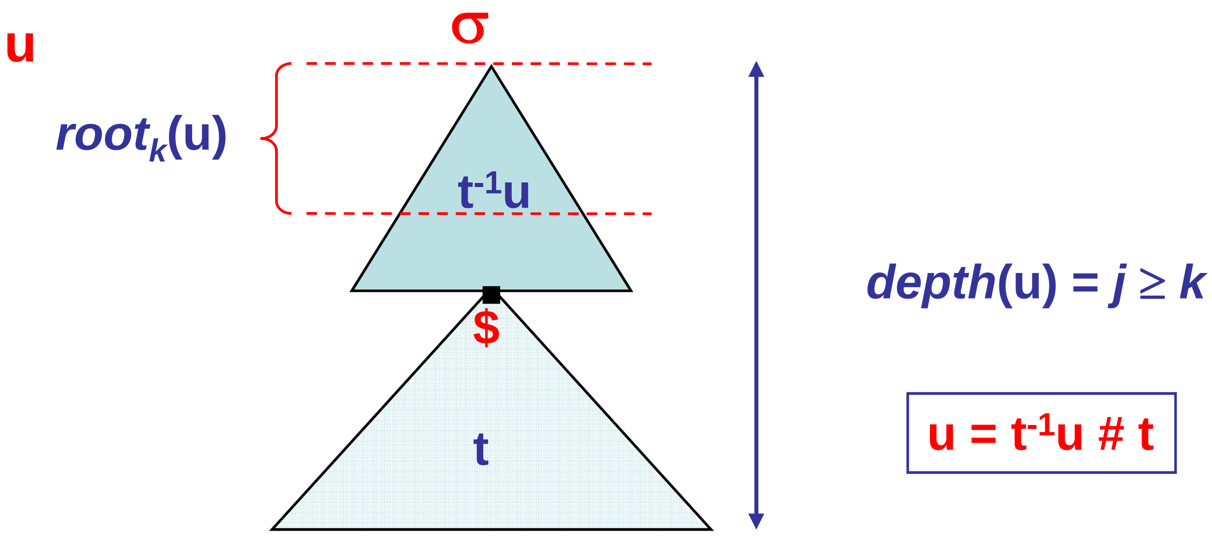
Multiset Tree Automata (MTA)

Any tree automaton A induces a multiset tree automaton MA such that if t ∈ L(A) then t ∈ L(MA)

For any tree automaton A, its induced multiset tree automaton MA accepts all the trees that A accepts together with all their "*mirrored*" trees



k-Testability in the Strict Sense (k-TSS)



$T \subseteq V^T$ is a k-TSS multiset tree language iff given two trees u_1 and u_2 in V^T with $root_{k-1}(u_1) = root_{k-1}(u_2)$, $u_1^{-1}T \neq \emptyset$ and $u_2^{-1}T \neq \emptyset$ implies that $u_1^{-1}T = u_2^{-1}T$

Let A be a multiset tree automaton over $V_\$^T$. Let u_1, u_2 in V^T be two trees such that $root_{k-1}(u_1) = root_{k-1}(u_2)$ and $t_1\# u_1$, $t_2\# u_2$ in L(A) for some valid contexts t_1 and t_2. If A is a k-TSS mirror tree automaton then $\delta(u_1) = \delta(u_2)$.

k-Testability in the Strict Sense (k-TSS)

Example I

A:

$\delta(\sigma, aa) = q_1$

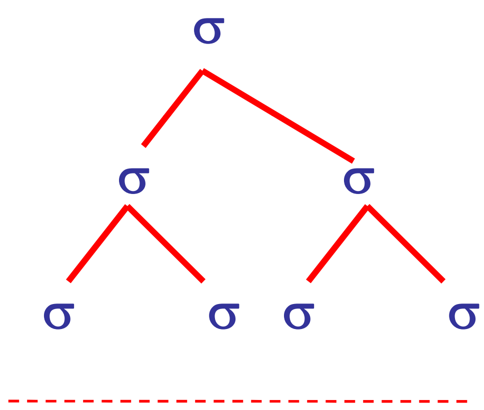
$\delta(\sigma, a) = q_2$

$\delta(\sigma, aq_2) = q_2$

$\delta(\sigma, q_1q_1) = q_1$

$\delta(\sigma, aq_2q_1) = q_3 \in F$

The MTA is k-TSS for any $k \geq 2$



Example II

A:

$\delta(\sigma, aa) = q_1$

$\delta(\sigma, bb) = q_2$

$\delta(\sigma, q_2q_2) = q_2$

$\delta(\sigma, q_1q_1) = q_1$

$\delta(\sigma, q_2q_1) = q_3 \in F$

The MTA is not k-TSS for any $k \geq 2$ (q_1, q_2 and q_3 share a common *k*-root)

k-reversability

Let $T \subseteq V^T$ and the integer value $k \geq 0$. **T is a k-reversible multiset tree language** if and only if, given whatever two trees u_1, u_2 in V^T such that $root_{k-1}(u_1) = root_{k-1}(u_2)$, whenever there exists a context t in $V_\$^T$ such that both $u_1\#\, t$, $u_2\#\, t$ in T, then $u_1^{-1}T = u_2^{-1}T$

Let A be a multiset tree automaton over $V_\$^T$. Let p_1, p_2 in Q be two states such that $root_k(L(p_1)) \cap root_k(L(p_2)) \neq \emptyset$. A is ***order k reset free*** if the automaton does not contain two transitions such that

$$\delta(\sigma, q_1 q_2 \ldots q_n p_1) = \delta(\sigma, q_1 q_2 \ldots q_n p_2)$$

Let A be a multiset tree automaton. **A is k-reversible** if A is order k reset free and for any two distinct final states f_1 and f_2 the condition

$$root_k(L(f_1)) \cap root_k(L(f_2)) \neq \emptyset$$

is fulfilled.

k-reversability

Example

A:

$\delta(\sigma, aa) = q_1$
$\delta(\sigma, a) = q_2$
$\delta(\sigma, q_2 q_2) = q_2$
$\delta(\sigma, aaq_1) = q_1$
$\delta(\sigma, aq_1 q_1) = q_3 \in F$
$\delta(\sigma, q_2 q_1) = q_3 \in F$

The MTA is not k-TSS for any $k \geq 2$.
The MTA is k-reversible for any $k \geq 2$.

Relations between k-reversible and k-TSS MT Languages

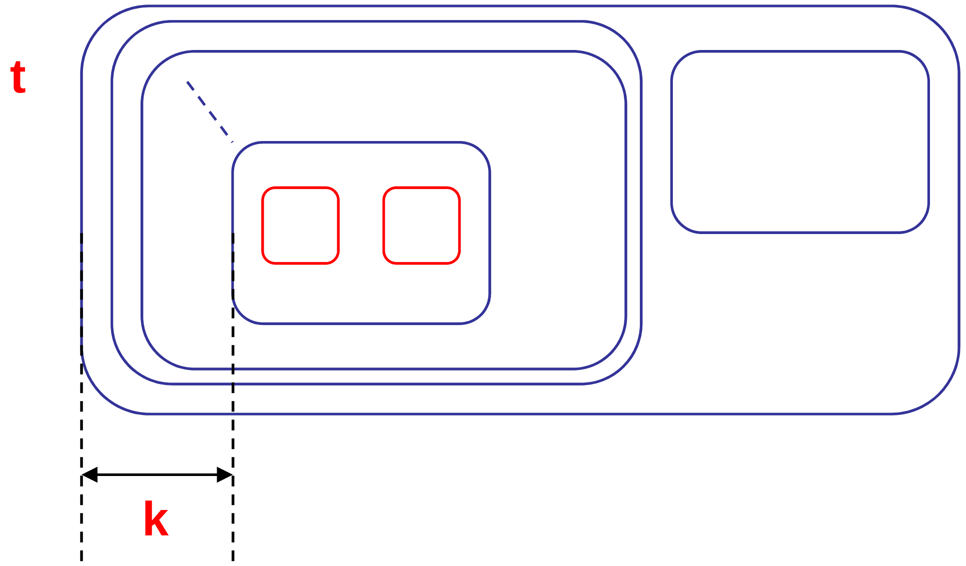
k-TSS forms an infinite hierarchy of MTL such that (k-1)-TSS languages are k-TSS languages

k-reversability forms an infinite hierarchy of MTL such that (k-1) reversible languages are k reversible languages

Theorem. Let $T \subseteq V^T$ and an integer value $k \geq 2$. If T is k-TSS then T is (k-1)-reversible.

From MTA to membrane structures

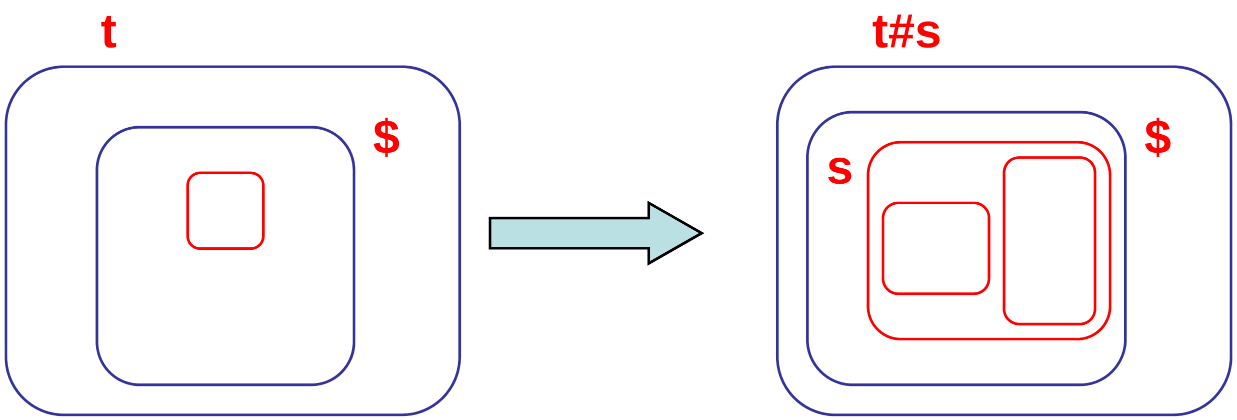
The meaning of $root_k(t)$



$root_k(t)$ takes into account all the membrane structures from the skin one up to a depth of order k (we croos k-1 membranes)

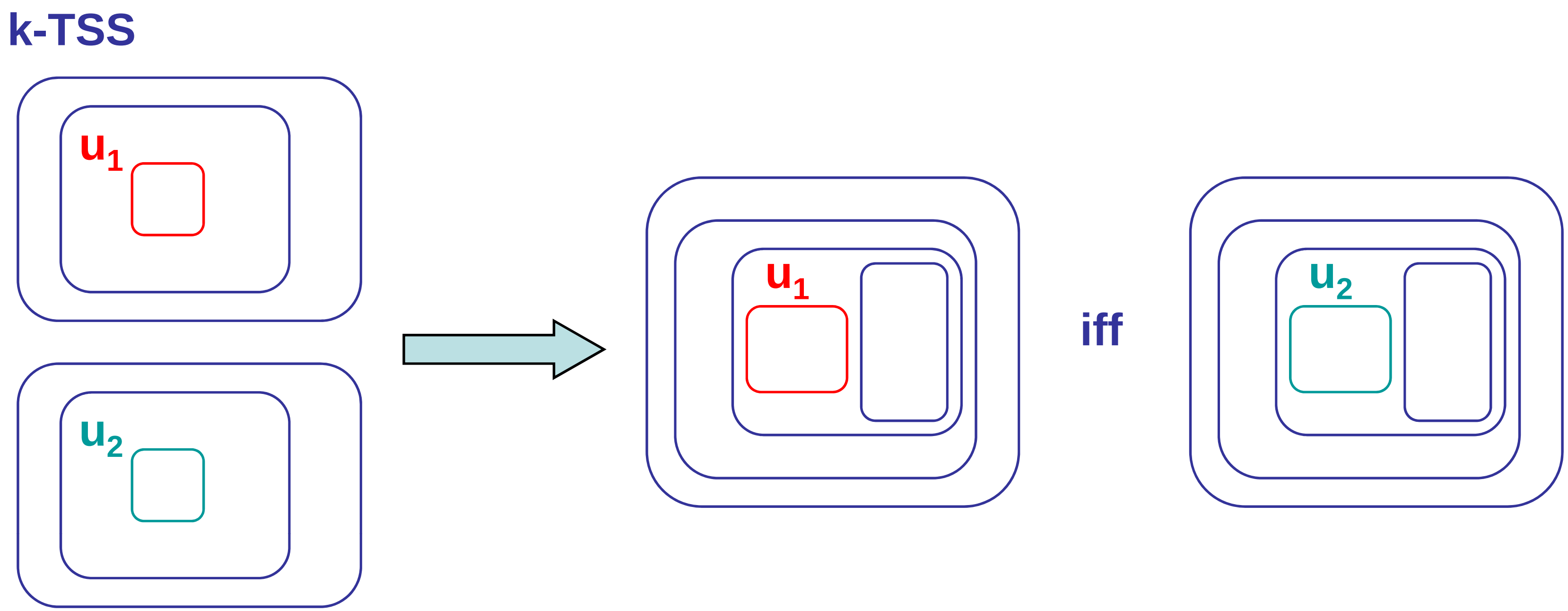
From MTA to membrane structures

The operator



The operator # allows new substructures by means of membrane creation at a given region (labelled by $)

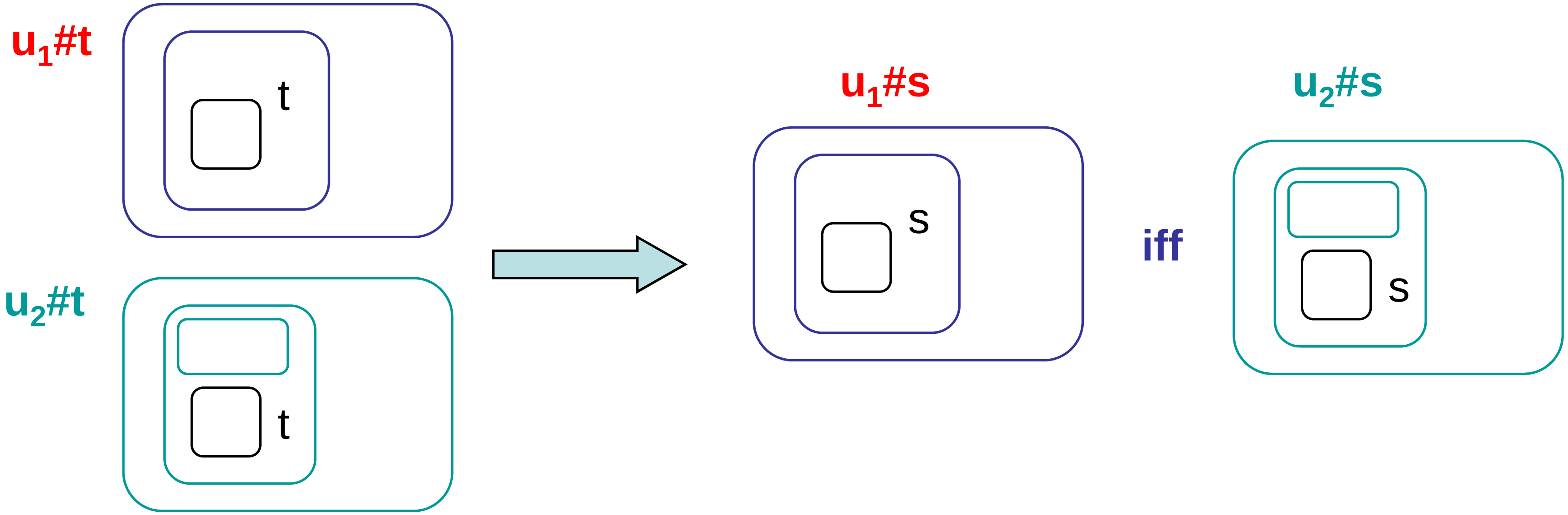
From MTA to membrane structures



k-TSS implies that, whenever we take two membrane structures u_1 and u_2 at any level of the P regions, if they are part of a structure that share a common substructure from the skin up to depth k-1 then they are part of the same set of structures (u_1 cannot appear as a substructure of a membrane structure if u_2 does not appear as a substructure of the same membrane structure at the same level).

From MTA to membrane structures

k- reversability



k-reversibility implies that whenever two membrane structures u_1 and u_2 share the same substructure up to length k-1, if u_1 and u_2 have a common structure t such that u_1#t and u_2#t are valid configurations of the P system, then u_1#s is a valid configuration of the P system iff u_2#s so is.

Conclusions

- We have introduced new classes of MT languages which can be defined by the transitions in their corresponding MTA.

- The features of k-TSS and k-reversability restrict the behavior of the P systems rules for membrane creation/deletion/duplication etc.

Future research

What is the set of integers/strings/vectors that P systems can generate/accept if their membrane structures are defined by (k) reversible/testable Multiset Tree Automata ?