Multigraphical Membrane Systems: a Visual Formalism for Modeling Complex Systems in Biology and Evolving Neural Networks

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Summary. A concept of a (directed) multigraphical membrane system, akin to membrane systems in [9] and [10], for modeling complex systems in biology, evolving neural networks, perception, and brain function is introduced.

1 Introduction

Statecharts described in [7] and their wide applications, including applications in system biology, cf. [6], and the formal foundations for natural reasoning in a visual mode presented in [11] challenge a prejudice against visualizations in exact sciences that they are heuristic tools and not valid elements of mathematical proofs.

We introduce a concept of a (directed) multigraphical membrane system to be applied for modeling complex systems in biology, evolving neural networks, perception, and brain function. A precise mathematical definition of this concept and its topological representation by Venn diagrams and the usual graph drawings constitute a kind of visual formalism related to that discussed in [7]. The concept of a multigraphical membrane system is some new variant of the notion of a membrane system in [9] and [10].

2 Multigraphical Membrane Systems

Membrane system in [9] and [10] are simply finite trees with nodes labeled by multisets, where the finite trees have a natural visual presentation by Venn diagrams.

We introduce (*directed*) *multigraphical membrane systems* to be finite trees with nodes labeled by (directed) multigraphs.

We consider directed multigraphical membrane systems of a special feature described formally in the following way.

A *sketch-like membrane system* $\mathcal{S}$ is given by:

- its *underlying tree* $\mathbb{T}_{\mathcal{S}}$ which is a finite graph given by the set $V(\mathbb{T}_{\mathcal{S}})$ of *vertices*, the set $E(\mathbb{T}_{\mathcal{S}}) \subseteq V(\mathbb{T}_{\mathcal{S}}) \times V(\mathbb{T}_{\mathcal{S}})$ of *edges*, and the *root* r which is a distinguished vertex such that for every vertex v different from r there exists a unique path from v into r in $\mathbb{T}_{\mathcal{S}}$, where for every vertex v we define $\mathrm{rel}(v) = \{v' \mid (v', v) \in E(\mathbb{T}_{\mathcal{S}})\}$ which is the set of vertices *immediately related* to v;
- its family $(G_v \mid v \in V(\mathbb{T}_{\mathcal{S}}))$ of finite directed multigraphs for G_v given by the set $V(G_v)$ of *vertices*, the set $E(G_v)$ of *edges*, the *source function* $s_v : E(G_v) \to V(G_v)$, and the *target function* $t_v : E(G_v) \to V(G_v)$ such that the following conditions hold:
  1) $V(G_v) = \{v\} \cup \mathrm{rel}(v)$,
  2) $E(G_v)$ is empty for every *elementary* vertex v, i.e. such that $\mathrm{rel}(v)$ is empty,
  3) for every *non-elementary* vertex v, i.e. such that $\mathrm{rel}(v)$ is a non-empty set, we have
     (i) $G_v(v, v')$ is empty for every $v' \in V(G_v)$,
     (ii) $G_v(v', v)$ is a one-element set for every $v' \in \mathrm{rel}(v)$,
     where $G_v(v_1, v_2) = \{e \in E(G_v) \mid s_v(e) = v_1 \text{ and } t_v(e) = v_2\}$.

For every non-elementary vertex v of $\mathbb{T}_{\mathcal{S}}$ we define:

- the *v-diagram* $\mathrm{Dg}(v)$ to be that directed multigraph which is the *restriction* of G_v to $\mathrm{rel}(v)$, i.e. $V(\mathrm{Dg}(v)) = \mathrm{rel}(v)$,

  $$E(\mathrm{Dg}(v)) = \big\{e \in E(G_v) \mid \{s_v(e), t_v(e)\} \subseteq \mathrm{rel}(v)\big\},$$

  the source and target functions of $\mathrm{Dg}(v)$ are the obvious restrictions of s_v, t_v to $E(\mathrm{Dg}(v))$, respectively,
- the *v-cocone* to be a family $(e_{v'} \mid v' \in \mathrm{rel}(v))$ of edges of G_v such that $s_v(e_{v'}) = v'$ and $t_v(e_{v'}) = v$ for every $v' \in \mathrm{rel}(v)$.

By a *model* of a sketch-like membrane system $\mathcal{S}$ in a category $\mathbb{C}$ with finite colimits we mean a family of graph homomorphisms $h_v : G_v \to \mathbb{C}$ (v is a non-elementary vertex of $\mathbb{T}_{\mathcal{S}}$) such that $h_v(v)$ is a colimit of the diagram $h_v \upharpoonright \mathrm{Dg}(v) : \mathrm{Dg}(v) \to \mathbb{C}$ and $(h_v(e_{v'}) \mid v' \in \mathrm{rel}(v))$ is a colimiting cocone for the v-cocone $(e_{v'} \mid v' \in \mathrm{rel}(v))$, where $h_v \upharpoonright \mathrm{Dg}(v)$ is the restriction of h_v to $\mathrm{Dg}(v)$.

For all categorical and sketch theoretical notions like graph homomorphism, colimit of the diagram, and colimiting cocone we refer the reader to [3].

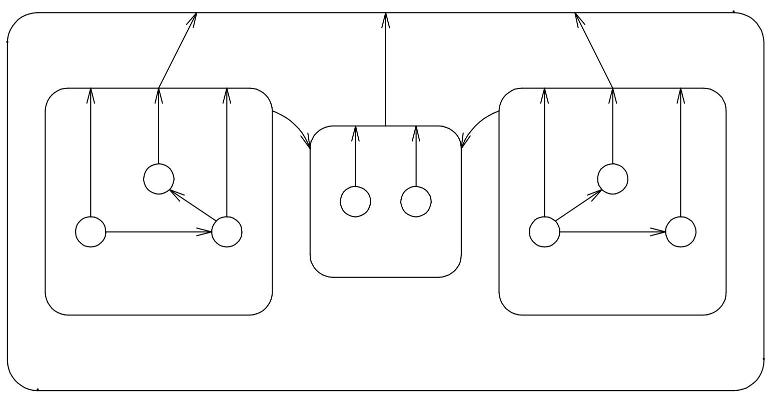
The idea of a sketch-like membrane system and its categorical model is a special case of the concept of a sketch and its model described in [3] and [8], where one finds that sketches can serve as a visual presentation of some data structure and data type algebraic specifications. On the other hand the idea of a sketch-like membrane system is a generalization of the notion of ramification used in [4] and [5] to investigate hierarchical categories with hierarchies determined by iterated colimits understood as in [4]. Hierarchical categories with hierarchies determined by iterated colimits are applied in [1] and [5] to describe various emergence phenomena in biology and general system theory. The iterated colimits identified with binding of patterns in neural net systems are expected in [5] to be applied in the investigations of binding problems in vision systems (associated with perception and brain function) in [12] and [13], hence the notion of sketch-like membrane system is aimed to be a tool for these investigations.

More precisely, sketch-like membrane systems are aimed to be presentations of objects of state categories of Memory Evolutive Systems in [4] and [5] similarly like strings of digits serve for presentation of numbers, where these state categories are hierarchical categories with hierarchies determined by iterated colimits. Hierarchical shape of sketch-like membrane systems and their categorical semantics reflect iterated colimit feature of objects of state categories of Memory Evolutive Systems.

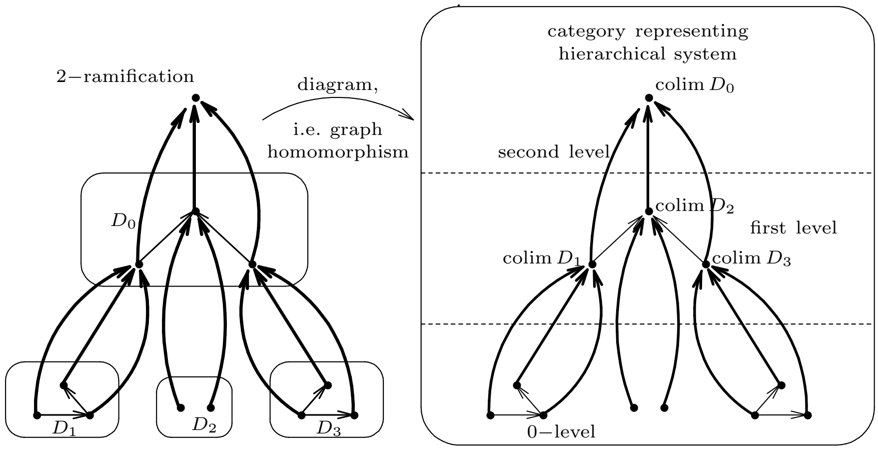
If we drop condition 3) in the definition of a sketch-like membrane system, we obtain these directed multigraphical membrane systems which appear useful to describe alternating organization of living systems discussed in [2] with a regard to nesting (represented by the underlying tree $\mathbb{T}_{\mathcal{S}}$) and interaction of levels of organization (represented by family of directed multigraphs G_v $(v \in V(\mathbb{T}_{\mathcal{S}}))$). According to [2] the edges in $G_v(v', v)$ describe integration, the edges in $G_v(v, v')$ describe regulation, and the edges of v-diagram $\mathrm{Dg}(v)$ describe interaction.

A directed multigraphical (a sketch-like) membrane system is illustrated in Fig. 1, whose semantics (model) in a hierarchical category is illustrated in Fig. 2.

Multigraphical membrane system corresponding to 2-ramification:



nodes—membranes, edges—objects,
neurons—membranes, synapses—objects.



the fat arrows are colimiting injections, i.e. the elements of colimiting cocones, respectively

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